

# Optimal Time Consistent Taxation with Overlapping Generations

Steve Ambler\*

CREFE, UQAM

preliminary: do not quote without permission

October 2001

## Abstract

The paper analyzes optimal time-consistent taxation in an overlapping generations model. The government must choose cohort-independent tax rates on capital and labor income as well as borrowing in order to finance an exogenous stream of expenditures. It cannot commit to future policies. We consider optimal feedback rules for tax rates and borrowing which are Markov-perfect and therefore time consistent by construction. Without precommitment, capital income tax rates are quantitatively significant in the long run, of the same order of magnitude as the share of public spending in GDP. We also analyze the optimal response of tax rates to economic shocks.

Key words: Optimal Fiscal Policy, Time Consistency, Overlapping Generations

JEL classification codes: E61, E62, C63

---

\*CREFE, UQAM, C.P. 8888, Succ. Centre-ville, Montréal, Québec, Canada H3C 3P8, phone (514) 987-3000 ext. 8372, fax (514) 987-8494, email [ambler.steven@uqam.ca](mailto:ambler.steven@uqam.ca). I would like to thank the CRSH and the Fonds FCAR for generous financial support and the European University Institute, where preliminary work on this project was undertaken, for its generous financial support and hospitality. The usual caveat applies.

# 1 Introduction

There is an extensive literature on optimal second-best taxation in dynamic general equilibrium (DGE) models.<sup>1</sup> With some exceptions, researchers have used models of representative agents with infinite planning horizons. Furthermore, it is common to assume that governments can precommit to future policies, so that time inconsistency is not a problem. Optimal policies are characterized as solutions to so-called *Ramsey problems* (1927), in which the government maximizes a social welfare function (typically just the utility function of the representative agent) subject to a set of constraints which ensure that private agents' first order conditions are satisfied. These models lead to fairly striking predictions concerning optimal tax rates, which are very different from the observed behavior of governments' fiscal policies.

These predictions include the following.<sup>2</sup> 1) In deterministic models, the optimal rate of taxation on capital income is zero in the long run. 2) With endogenous growth and human capital, all capital tax rates are zero in the long run. 3) The first-best social optimum can be achieved if it is possible to tax capital income at arbitrarily high rates (a capital levy) in the short run, while capital is supplied inelastically. To make the problem of optimal taxation a non-trivial one, arbitrary limits are imposed on capital income tax rates at the beginning of the government's optimal program. 4) The

---

<sup>1</sup>The possibility of lump-sum taxation is excluded in these models, so that the first-best social optimum cannot be attained.

<sup>2</sup>See Ambler (1999), Chari, Christiano and Kehoe (1995), Chari and Kehoe (1998), and Judd (1999) for surveys of the literature and of the standard results.

government's optimal plan is not time consistent. If the government can re-optimize and is not restricted in its choice of capital income tax rates, it can achieve the first-best outcome from that point onward with a capital levy.<sup>3</sup> 5) In stochastic versions of the problem, the *average* tax rate on capital income converges to zero, but it is optimal to vary tax rates on income from financial assets a lot in the short run in response to shocks.

The divergence between the predictions of representative agent models of optimal taxation and the data means that these models are poor as positive models of government behavior. Either real-world governments have behavior which is highly suboptimal,<sup>4</sup> or the representative agent approach fails to capture some essential features of the optimal policy problem. As normative models, they have not satisfactorily addressed the issue of the credibility of precommitment. This has recently led researchers to examine optimal fiscal policy in environments richer than representative agent models with government precommitment. Some of this literature is surveyed in the second section of the paper.

This paper sets up a model of overlapping generations with very different

---

<sup>3</sup>For this reason, I would argue that explicit mechanisms such as trigger strategies to support the precommitment equilibrium are implausible, since they must lead to outcomes that are worse than the first-best outcome. Papers which discuss commitment mechanisms in detail, such as Stokey (1993) and Chari and Kehoe (1993) use models in which the attainment of the first-best outcome is excluded by construction.

<sup>4</sup>I refer here to the qualitative and quantitative properties of the tax rates themselves. Many studies have shown that rule of thumb behavior by private agents in dynamic models can lead to very different behavior, while it does not necessarily cause large welfare losses. It may also be the case that observed government behavior is suboptimal but not very costly in terms of social welfare.

predictions. Capital ownership is not uniform across cohorts since households are born without assets and accumulate capital as they grow older. In this setting, very high capital income tax rates are not optimal,<sup>5</sup> since they redistribute too much income away from the old.<sup>6</sup> Since the government cannot precommit to future rates of taxation, capital income tax rates do not converge to zero in the long run. With precommitment, the government could achieve its distributional goals by taxing (exclusively or primarily) each cohort's labor income.<sup>7</sup> I use linear-quadratic approximation techniques are used to derive the optimal behavior of private households for a given government feedback rule, and iterative techniques to derive the government's optimal feedback rule.<sup>8</sup>

The paper is structured as follows. The following section briefly reviews some of the relevant literature on optimal taxation in macroeconomic models. The model is presented in the third section, along with a description of the government's optimal policy problem. The model's calibration and the simulation results are presented in the fourth section. The fifth section concludes.

---

<sup>5</sup>Age-dependent tax rates are excluded by assumption.

<sup>6</sup>This means that it is not necessary to impose arbitrary restrictions on rates of capital income taxation in the very short run when setting up the problem.

<sup>7</sup>This is essentially the result shown by Erosa and Gervais (1998), which I discuss in the next section.

<sup>8</sup>I conjecture that one of the reasons that the use of Ramsey problems has been so popular in the literature is that they allow researchers to use the so-called *primal approach* (see Chari and Kehoe, 1998, for details), which simplifies the calculation of optimal policies. Using this approach, it is possible to solve directly for the optimal allocations in the economy. The tax rates and prices that support these allocations can be derived once these allocations are known.

## 2 Relevant Literature

The classic result on the convergence of capital income tax rates to zero was first derived by Chamley (1985, 1986) and Judd (1985, 1987), using non-stochastic versions of the neoclassical growth model and extending the work of Arrow and Kurz (1970) and Ramsey (1928). Optimal policy in stochastic models was studied by Zhu (1992) and in a series of papers by Chari, Christiano and Kehoe (1991, 1994, 1995). Jones, Manuelli and Rossi (1993, 1997) analyzed optimal capital income taxation in endogenous growth models with both physical and human capital.

The intuition of the classic result is straightforward. Representative agents models with perfect competition have Euler equations for the marginal utility of private consumption of the form (ignoring uncertainty):

$$\lambda_t = \beta F'(k_{t+1})(1 - \tau_{t+1}^k)\lambda_{t+1}, \quad (1)$$

where  $\lambda_t$  is the representative agent's marginal utility of consumption at time  $t$ ,  $\beta$  is the subjective discount rate,  $F'(k_{t+1})$  is the private rate of return on capital, which under perfect competition is just the marginal productivity of capital (the value of the marginal unit of capital left over at the end of  $t + 1$  in the interests of simplicity), and  $\tau_{t+1}^k$  is the rate of taxation on capital income in  $t + 1$ .

Using the *primal approach* (see footnote (4) above), the government's first order condition for its choice of the time  $t + 1$  capital stock leads to an

equation of the form

$$\mu_t = \beta F'(k_{t+1})\mu_{t+1}, \quad (2)$$

where  $\mu_t$  is the *social* marginal utility of consumption at time  $t$ . In the long run, both the private and social marginal utilities of consumption are constant, and it follows immediately that the capital income tax rate should be zero. Judd (1999) derives the result with less restrictive assumptions. He shows that it is not even necessary to suppose that the economy converges to a steady state. The flatness of the representative agent's intertemporal consumption profile is crucial in deriving the result.

Models in which long-run capital income tax rates are not zero have started to appear in the literature recently. Benhabib and Rustichini (1997) and Benhabib, Rustichini and Velasco (1996) study optimal taxation in representative agent models without commitment. They consider optimal government strategies which are robust at time  $t$  to any possible deviations from time  $t$  on, in the sense that any deviation worsens social welfare.<sup>9</sup> They derive the result that, in order to create the necessary incentives for the government not to deviate from its optimal plan, capital income taxation rates must be strongly *negative*. Klein and Rios-Rull (1999) study a representative agent model in which the government must balance its budget each period without borrowing. Optimal capital income taxation rates are nonzero in the long run, and no additional restriction is necessary on initial capital income

---

<sup>9</sup>As in representative agent models of optimal taxation with commitment, they impose a limit on the rate of taxation of capital income at the start of the optimal program

tax rates, since the government is prohibited at all times from building up arbitrarily large claims on the private sector. The stochastic properties of tax rates in their model are closer to the data than in similar models with unlimited government borrowing.

In an important recent paper, Erosa and Gervais (1998) consider a model with overlapping generations of households with arbitrary, finite planning horizons. They show analytically that optimal fiscal policy with commitment involves a tax rate on capital income that does not generally converge to zero in the long run, in contrast to most representative agent models. However, when preferences are additively separable in consumption and leisure, optimal capital income tax rates converge to zero after an initial period when they are positive. Atkeson, Chari and Kehoe (1999) demonstrate a similar result. In addition, in calibrated versions of their model, Erosa and Gervais show that capital income tax rates are quantitatively quite small in the long run (less than three percent).

Overlapping generations models have also been used to examine the effects of exogenous (not satisfying explicit optimality conditions) changes in fiscal policy on economic equilibrium and welfare. These models have included overlapping generations models in which agents have finite horizons (Auerbach and Kotlikoff, 1987; Ventura, 1996), and models which use the Yaari (1965) assumption of a constant probability of death or the Weil (1989) assumption of overlapping generations of agents with infinite horizons (James, 1994; Scarth, 1999).

### 3 The Model

The economy consists of overlapping generations of households with planning horizons of  $I$  periods.<sup>10</sup> The total population of the economy is constant.<sup>11</sup> There are no bequests, and individuals are born without assets. Perfectly competitive firms rent labor and capital from households and produce goods which are sold to both households and to the government. The government finances an exogenous stream of expenditures by taxing labor and capital income and by borrowing. Its exogenous expenditures include age-dependent transfers which can be calibrated to mimic features of social security systems.

#### 3.1 Households

Individual households born at date  $t$  maximize the lifetime utility function given by

$$U_{0,t} = E_t \sum_{i=0}^I \beta^i u(c_{i,t+i}, 1 - n_{i,t+i}) \quad (3)$$

where  $c_{i,t+i}$  is consumption of households of age  $i$  in period  $t + i$ , and  $n_{i,t+i}$  is hours worked by households of age  $i$  in period  $t + i$ . Households of age  $i$  have the following period budget constraint in period  $t$ :

$$\epsilon_i w_t (1 - \tau_t) n_{i,t} + (1 + r_t (1 - \tau_t^k)) a_{i,t} + s_{i,t} = c_{i,t} + a_{i+1,t+1} \quad (4)$$

where  $\epsilon_i$  is an age-specific productivity parameter,  $w_t$  is the real wage per efficiency unit of labor,  $\tau_t$  is the tax rate on labor income,  $a_{i,t}$  is asset holdings

---

<sup>10</sup>The goal is calibration to annual data.

<sup>11</sup>The specification of the model could easily be modified to incorporate realistic demographic dynamics. This would not alter the main conclusion of the paper.

at the beginning of period  $t$ ,  $r_t$  is the rate of return on assets,  $\tau_t^k$  is the rate of taxation on asset income,  $s_{i,t}$  denotes social security transfers to households of age  $i$ , and  $c_{i,t}$  is consumption of households of age  $i$  at time  $t$ . Age-specific tax rates are ruled out by assumption, but age-specific social security transfers are allowed. Households are born without assets and die without debts, so that

$$a_{0,t} = a_{I+1,t} = 0, \quad \forall t.$$

The asset market structure of the economy is as follows. All assets pay the same rate of return as the rental rate on capital, and are taxed at the same rate. All lending and borrowing across cohorts takes in the form of this type of asset, as does borrowing by the government. If  $A_{i,t}$  represents average asset holdings by the cohort of age  $i$  and if  $b_t$  represents the stock of government bonds at time  $t$ , in equilibrium it will be the case that

$$\left( \sum_{i=1}^I A_{i,t} \right) - B_t \equiv A_t - B_t = K_t,$$

where  $K_t$  is the aggregate capital stock. In equilibrium it will also be the case that

$$r_t + \delta = R_t$$

where  $R_t$  is the competitive rental rate of capital. This asset structure simplifies greatly the solution of the model.<sup>12</sup>

---

<sup>12</sup>Rios-Rull (1999) shows that that enriching this type of model with state-contingent markets makes little quantitative difference.

Period utility is given by

$$u(c_{i,t+i}, 1 - n_{i,t+i}) = \frac{(c_{i,t}^\theta (1 - n_{i,t})^{1-\theta})^{1-\sigma}}{1 - \sigma}, \quad (5)$$

where  $0 < \theta < 1$  and  $\sigma > 0$  captures both the intertemporal elasticity of substitution and relative risk aversion. Households' first order conditions for utility maximization include the following:<sup>13</sup>

$$\frac{\partial u_{i,t}}{\partial c_{i,t}} - \lambda_{i,t} = 0, \quad (6)$$

$$-\frac{\partial u_{i,t}}{\partial n_{i,t}} + \lambda_{i,t} \epsilon_i w_t (1 - \tau_t) = 0, \quad (7)$$

$$-\lambda_{i,t} + E_t \left( \beta \left( 1 + r_{t+1} \left( 1 - \tau_{t+1}^k \right) \right) \lambda_{i+1,t+1} \right) = 0. \quad (8)$$

The first two first order conditions hold for  $i = 0, 1, \dots, I$  while the last first order condition holds only for  $i = 0, 1, \dots, I - 1$ , since of course it must be the case that

$$\lambda_{I+1,t+1} = 0.$$

The consumption Euler equation and labor supply equation can easily be used to solve for the steady-state life cycle profile of consumption and labor supply given the steady-state real wage and capital rental rate, by adjusting the initial level of consumption, iterating forward, and verifying if the lifetime budget constraint is satisfied.

We could model retirement in this economy by setting  $\epsilon_i$  to a very low value for older cohorts, with publicly-provided old age pensions captured by

---

<sup>13</sup>For the purposes of numerical simulations, we derive households' optimal feedback rules using dynamic programming techniques, which give feedback rules depending only on the economy's predetermined state variables.

the  $s_{i,t}$  terms. Since we simulate the model by using linear-quadratic approximation techniques, in practice this means that the simulation algorithm must be adapted in order to impose the constraint that hours worked by retired households are zero.<sup>14</sup>

### 3.2 Firms

A representative competitive firm rents labor from young agents and capital from old agents and produces output according to a standard aggregate production function given by

$$y_t = z_t N_t^\alpha K_t^{(1-\alpha)}, \quad (9)$$

where

$$N_t = \sum_{i=0}^I \epsilon_i n_{i,t}$$

Profit maximization leads to the following standard optimality conditions which determine the real wage rate and the rental rate of capital:

$$w_t = \alpha z_t N_t^{(\alpha-1)} K_t^{(1-\alpha)}, \quad (10)$$

$$R_t = (1 - \alpha) z_t N_t^\alpha K_t^{-\alpha}. \quad (11)$$

The log of the level of technology obeys the following law of motion:<sup>15</sup>

$$\ln(z_t) = \ln(\bar{z}) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt}, \quad (12)$$

---

<sup>14</sup>The only choice variable for retired households is consumption, which has to satisfy the consumption Euler equation and the endpoint condition that assets are drawn down to zero at death. Given the steady-state interest rate, we can impose the value of assets at retirement that satisfies these restrictions, and solve the model backwards for the behavior of younger cohorts in the steady state.

<sup>15</sup>We could easily allow for a trend in technology, but this would complicate the algebra without changing the nature of the results.

where  $\varepsilon_{zt}$  is a white noise shock.

### 3.3 The Government

The government finances an exogenous stream of spending and social security payments by taxing capital and labor income and buy borrowing. Its expenditures are given by

$$\ln G_t = (1 - \rho_g) \ln G + \rho_g \ln G_{t-1} + \varepsilon_{g,t}, \quad (13)$$

where  $G_t$  is government spending on goods and services,  $G$  is the long run level of government spending, and  $\varepsilon_{g,t}$  is an unpredictable spending shock.

The government's flow budget constraint is given by

$$G_t + (1 + r_t)B_t + S_t = w_t N_t \tau_t + r_t A_t \tau_t^k + B_{t+1}, \quad (14)$$

where

$$S_t \equiv \sum_{i=0}^I s_{i,t}, \quad (15)$$

and where  $B_t$  is the stock of government debt. We abstract from other forms of taxation such as consumption taxes. In a slightly different context, Judd (1999) shows that adding consumption taxes does not give the government an independent policy instrument. It would imply that several different combinations of tax rates could be used to achieve the same competitive equilibrium. We ignore this possibility in the interests of simplicity. In order to simplify the algebra, the capital tax rate is applied uniformly to all assets including government debt.

The government's social welfare function is given by

$$SW_t = \sum_{i=0}^{\infty} \tilde{\beta}^{-I+i} U_{0,t-I+i} \quad (16)$$

This is just the sum of lifetime utilities of households discounted to their dates of birth, using the social discount rate  $\tilde{\beta}$ . Calvo and Obstfeld (1988) show that the utility of individual cohorts must be discounted relative to the beginnings of their lives in order to avoid problems of time inconsistency arising *solely from the specification of the government's preferences* as opposed to time inconsistency arising from constraints imposed on the financing of the government's activities.<sup>16</sup> In the calibration of the model I set the social discount rate  $\tilde{\beta}$  equal to private agents' discount rate, but in principle the two could be different. Many of the terms in this infinite sum involve consumption and hours at dates  $t - 1$  or earlier. These terms act as constants that shift the level of the social welfare function without changing the government's optimal policy.

The terms in the social welfare function can be regrouped in order to facilitate writing the government's problem as a Bellman equation. We have

$$SW_t = \sum_{i=0}^I \sum_{j=0}^{\infty} \quad (17)$$

This equation shows that, if the social discount rate is equal to private agents' discount rates, the government's one-period return function, which is just the period- $t$  term from the social welfare function, is just the algebraic sum of cohorts' period- $t$  utilities across cohorts.

---

<sup>16</sup>See Ambler and Desruelle (1991) for a discussion of how time inconsistency arises in dynamic games when agents share the same objectives but face different constraints.

### 3.4 Equilibrium

Competitive equilibrium in the model is defined in the standard way. Private agents maximize their utilities for given government policy rules and given prices. Markets clear given these prices. In addition, we impose the requirement that the government's policy rules maximize social welfare subject to the laws of motion of the economy which are derived by imposing aggregate consistency constraints on individual agents' optimality conditions. Aggregate consistency constraints stipulate that individual agents' choice values such as future holdings of financial assets and capital, hours worked, consumption, etc. are equal in equilibrium to their per capita aggregate counterparts.

The model's dynamics can be reduced to a system of the form:

$$S_{t+1} = S(Z_t, S_t, u_t), \quad (18)$$

where  $S_t$  is a vector of endogenous state variables at time  $t$ ,  $Z_t$  is a vector of exogenous state variables, and  $u_t$  is a vector of government policy variables.

The exogenous state vector is given by

$$Z_t = \{1, z_t\}.$$

The vector includes a constant term in order to facilitate the numerical solution of the model. The vector of endogenous state variables is

$$S_t = \{A_t, b_t\},$$

where  $A_t$  is the  $(I+1) \times 1$  vector of per-capita asset holdings of the  $I+1$  cohorts of households, and  $b_t$  is equal to total private-sector holdings of government debt. The aggregate capital stock in the economy is given by

$$K_t = \left( \sum_{i=1}^I A_{it} \right) - b_t.$$

The vector of government policy variables is

$$u_t = \{\tau_t, B_{t+1}\}.$$

One of the laws of motion for the state variables is just

$$b_{t+1} = B_{t+1},$$

which has the effect of imposing the equilibrium condition that net bond holdings by the private sector are equal to the supply of bonds issued by the government. By using dynamic programming to derive agents' feedback rules, the model's dynamics can be written in a form which does not depend on forward-looking relative prices. This makes the government's optimal policy problem recursive, so that standard dynamic programming techniques can be used to solve it.

### 3.5 Optimal Government Policies

Following Ambler and Cardia (1997) and Ambler and Paquet (1996, 1997), the government solves its optimal policy problem using dynamic programming methods and assumes that all future governments will do the same. Klein and Rios-Rull (1999) use a similar formulation of the government's

optimal policy problem in the context of a representative agent model with a period-by-period budget constraint. As they note, this approach can trace its origins to Kydland and Prescott (1980). By construction, the government's and private agents policy rules depend only on the current state of the economy. Equilibrium in the model is sometimes known as Markov-perfect equilibrium: see Bernheim and Ray (1989) and Maskin and Tirole (1993).

The optimal policy problem does not have an analytical solution. We find an approximate numerical solution to the problem using a quadratic approximation to the government's social welfare function given by equation (16) and a linear approximation to the economy's laws of motion as described in equation (18). The pseudo-algorithm used to solve the model is described in an appendix. The *GAUSS* code for the algorithm itself is available on request from the author.

### **3.6 Existence and Uniqueness**

For given policy rules, It is possible to demonstrate the existence of a competitive equilibrium in this economy using a proof similar to that in Rios-Rull (1996) and Balasko, Cass and Shell (1980). The uniqueness of the government's Markov-perfect strategy is a more delicate issue. [Section incomplete]

## 4 Calibration and Simulation Results

Preliminary results are presented based on a calibration with  $I = 2$ . Agents work in the first period of their lives. Young agents have only wage income, while old agents have only capital income, which brings out the distributional consequences of labor and capital income taxation in a very stark way. The utility function of the cohort born at time  $t$  is given by:

$$U_t = \frac{1}{1-\sigma} (c_{yt} - \gamma n_t^\theta)^{(1-\sigma)} + \beta \frac{1}{1-\sigma} (c_{o,t+1})^{(1-\sigma)},$$

Table 1 summarizes the base-case parameter values used to generate the numerical simulation results. Table 2 summarizes the model's steady state properties. The model's predictions concerning the response of tax rates to fluctuations in the level of technology are given in Table 3, along with predictions from the model of Klein and Rios-Rull (1999) and the empirical behavior of U.S. tax rates.

The optimal steady state tax rates on labor and capital income are respectively equal to 0.165 and 0.203. The capital tax rate is quantitatively important, and exceeds the ratio of government spending to output, which is calibrated to equal 0.20. Total tax revenues in the steady state are insufficient to finance government spending. In order to finance the difference between spending on goods and services and tax revenue, the government must receive a positive income from its holdings of assets. The optimal level of government debt in the long run is equal to -0.008, which is approximately equal to 6% of steady state real output. The steady state labor and capital

income tax rates are both below the average marginal tax rates in the U.S. data (see Table 3), although government expenditure is calibrated to approximately match its share in output. This can be explained by the negative level of government debt in the steady state in the model.

Table 2 also gives the equilibrium level of output in the steady state, and the decomposition of aggregate demand into consumption, investment, and government spending on goods and services. The ratios of consumption, output, and investment to output are not far from those in the data for industrialized economies. The capital to output ratio in annualized terms ( $25k/y$ ) is equal to 5.22. The annualized real rate of interest on bonds ( $(1+r)^{(1/25)} - 1$ ) is equal to 0.013. The capital rental rate gives the same after-tax rate of return from financial bonds and physical capital. We have  $\beta(1+r) = 1.033$  in the steady state, which implies that there is a slightly upward-sloping profile for the marginal utility of consumption for each cohort of agents.

The optimal feedback rule for the government is given by:

$$\begin{aligned} \begin{bmatrix} \tau_{nt} \\ B_{t+1} \end{bmatrix} &= \begin{bmatrix} 0.017 & 0.391 \\ 0.010 & -0.047 \end{bmatrix} \begin{bmatrix} 1 \\ z_t \end{bmatrix} \\ &+ \begin{bmatrix} 5.139 & 0.001 & 0.000 \\ -0.635 & 0.000 & 0.000 \end{bmatrix} \begin{bmatrix} k_t \\ b_t \\ r_t \end{bmatrix} \end{aligned} \quad (19)$$

The optimal labor income tax rate responds positively to both technology shocks and the level of the capital stock, while the level of government debt responds negatively to both of these variables.

Figure 1 gives the response of the labor income and capital income tax

rates to a positive one-standard-deviation technology shock. As can be inferred from the signs of the coefficient matrices of the feedback rule (19), the labor income tax rate increases in response to a positive technology shock and then decreases gradually to its steady state level. The optimal capital income tax rate jumps up when the shock hits, and then decreases sharply in the following period, approaching its steady state level from below. The responses of optimal tax rates to technology shocks are qualitatively similar to those obtained by Klein and Rios-Rull (1999) in a representative-agent model of time consistent taxation without borrowing,<sup>17</sup> except that they do not obtain an initial positive response of the capital income tax rate. The difference is explained by the fact that Klein and Rios-Rull assume that the government can commit to its (state-contingent) capital income tax rate at least for the following period.<sup>18</sup>

The first column in Table 3 shows the variance, standard deviations and the coefficients of variation of capital and labor income tax rates for the model's base-case parameter values.<sup>19</sup> We compare our results to those in Klein and Rios-Rull (1999). The second column in the table shows results on the volatility of tax rates in their representative-agent model with com-

---

<sup>17</sup>They exclude the possibility of borrowing. As we note in the introduction, allowing the government to set the future level of government debt (including a *negative* level of debt) would allow it to attain the first best social optimum.

<sup>18</sup>There is also a quantitative difference. Klein and Rios-Rull show results for stochastic simulations in which the stochastic process governing technology is a two-state Markov process.

<sup>19</sup>The coefficients of variation are just the absolute values of the ratios of the standard deviations to the means.

mitment by the government, and the last column shows results for their model without commitment, both for the case where the period length is one year. The mean capital and labor income tax rates in our model are close to those in the data, and closer to their values in the data than those predicted by the Klein and Rios-Rull model, except in the case of the labor income tax rate with no commitment. The standard deviations of tax rates in our model are quite close to those in the U.S. data, and close to those in Klein and Rios-Rull for the version of their model without commitment. Their results show that optimal capital income tax rates in models with precommitment are highly variable compared to labor income tax rates. The latter are extremely smooth when the government can precommit to its optimal policy. This is a standard feature of models of optimal taxation with commitment. As summarized by Klein and Rios-Rull (p.20), “under commitment the burden of taxation is borne almost completely by labor while capital taxation accommodates all surprises.” In representative agent models of optimal taxation with government borrowing, such as Chari, Christiano and Kehoe (1994), capital income tax rates are generally even more volatile.

## 5 Conclusions

The paper develops an overlapping generations model in order to analyze optimal fiscal policy in the absence of commitment by the fiscal authorities. Optimal policies are Markov-perfect by construction.

The main result of the analysis is that optimal capital income tax rates

are much higher than in previous studies on optimal taxation in dynamic general equilibrium models, and closer to the average marginal tax rates on capital income in industrialized countries. We do not need to impose arbitrary limits on capital income tax rates in the short run in order to avoid the capital levy problem. The model predicts an optimal capital income tax rate which is much more volatile than that of the optimal labor income tax rate. The relative volatility of capital income taxes is lower than in representative agent models of optimal taxation with precommitment, but it is still higher than in the data for industrialized countries. The model also predicts, for a wide range of parameter values, that the optimal level of government debt in the steady state is negative. This is perhaps the major divergence between the predictions of the model and the data. Aiyagari and McGrattan (1994) show that the optimal level of government debt in a model of heterogeneous agents subject to liquidity constraints is positive. The availability of government bonds allows liquidity-constrained agents greater consumption-smoothing possibilities. Their model one of the few in the literature capable of rationalizing a positive level of debt as the outcome of maximizing behavior by the government. Adding liquidity constraints to our model would be the most obvious way to have optimal debt levels match the data more closely.

It would clearly be interesting to endogenize at least a component of the government spending decision. This would involve modelling government spending as partially productive and possibly modelling the public good na-

ture of certain types of government spending such as military spending. It would also be possible to use the framework developed here to analyze optimal social security systems (subject to realistic institutional constraints). Finally, an interesting extension would be to analyze the impact of optimal fiscal policy on growth in an endogenous growth model. These extensions are reserved for future work.

## References

- Aiyagari, Rao and Ellen McGrattan (1995), "The Optimum Quantity of Debt", Staff Report 203, Federal Reserve Bank of Minneapolis
- Ambler, Steve (1999), "Les modèles à agent représentatif et la politique de taxation optimale", *Actualité économique: Revue d'analyse économique* 75, 539-557. Available as cahier de recherche 91, Center for Research on Economic Fluctuations and Employment, Université du Québec à Montréal  
<http://ideas.uqam.ca/ideas/data/Papers/crecrefwp91.html>
- Ambler, Steve (2000), "Optimal Time Consistent Taxation with Overlapping Generations", cahier de recherche 111, Centre de recherche sur l'emploi et les fluctuations économiques, UQAM, 1999  
<http://ideas.uqam.ca/ideas/data/Papers/crecrefwp111.html>
- Ambler, Steve and Emanuela Cardia (1997), "Optimal Government Spending in a Business Cycle Model", *Business Cycles and Macroeconomic Stability: Should We Rebuild Built-In Stabilizers?* Jean-Olivier Hairault, Pierre-Yves Hénin and Franck Portier, editors, Kluwer Academic Press, 31-53
- Ambler, Steve and Dominique Desruelle (1991), "Time Inconsistency in Time-Dependent Team Games", *Economics Letters* 37, 1-6
- Ambler, Steve and Alain Paquet (1996), "Fiscal Spending Shocks, Endogenous Government Spending and Real Business Cycles", *Journal of Economic Dynamics and Control* 20, 237-256
- Ambler, Steve and Alain Paquet (1997), "Recursive Methods for Computing Equilibria of General Equilibrium Dynamic Stackelberg Games", *Economic Modelling* 14, 155-173
- Arrow, Kenneth J. and Mordecai Kurz (1970), *Public Investment, the Rate of Return, and Optimal Fiscal Policy*. Baltimore, Johns Hopkins
- Atkeson, Andrew, V.V. Chari and Patrick Kehoe (1999), "Taxing Capital Income: A Bad Idea", *Federal Reserve Bank of Minneapolis Quarterly Review* 23, 3-17

- Auerbach, Alan and Laurence Kotlikoff (1987), *Dynamic Fiscal Policy*. Cambridge, Cambridge University Press
- Balasko, Yves, David Cass and Karl Shell (1980), "Existence of Competitive Equilibrium in a General Overlapping-Generations Model", *Journal of Economic Theory* 23, 307-322
- Benhabib, Jess and Aldo Rustichini (1997), "Optimal Taxes without Commitment", *Journal of Economic Theory* 77, 231-259
- Benhabib, Jess, Aldo Rustichini and Andrés Velasco (1996), "Public Capital and Optimal Taxes without Commitment", C.V. Starr Center working paper 96-19, New York University
- Bernheim, B. Douglas and Debraj Ray (1989), "Markov Perfect Equilibria in Altruistic Growth Economies with Production Uncertainty", *Journal of Economic Theory* 47, 195-202
- Calvo, Guillermo and Maurice Obstfeld (1988), "Optimal Time-Consistent Fiscal Policy with Finite Lifetimes: Analysis and Extensions" in Elhanan Helpman, Assaf Razin and Efraim Sadka, eds., *Economic Effects of the Government Budget*. Cambridge, MA, MIT Press
- Chamley, C.P. (1985), "Efficient Taxation in a Stylized Model of Intertemporal General Equilibrium", *International Economic Review* 26, 451-468
- Chamley, C.P. (1986), "Optimal Taxation of Income in General Equilibrium with Infinite Lives", *Econometrica* 54, 607-622
- Chari, V.V., Lawrence J. Christiano and Patrick J. Kehoe (1991), "Optimal Fiscal and Monetary Policy: Some Recent Results", *Journal of Money, Credit and Banking* 23, 519-539
- Chari, V.V., Lawrence J. Christiano and Patrick J. Kehoe (1994), "Optimal Fiscal Policy in a Business Cycle Model", *Journal of Political Economy* 102, 617-652
- Chari, V.V., Lawrence J. Christiano and Patrick J. Kehoe (1995), "Policy Analysis in Business Cycle Models" in Thomas F. Cooley, ed., *Frontiers of Business Cycle Research*. Princeton, Princeton University Press
- Chari, V.V. and Patrick J. Kehoe (1993), "Sustainable Plans and Debt", *Journal of Economic Theory* 61, 230-261
- Chari, V.V. and Patrick J. Kehoe (1998), "Optimal Fiscal and Monetary Policy", Staff Report 251, Federal Reserve Bank of Minneapolis, July
- Erosa, Andrés and Martin Gervais (1998), "Optimal Taxation in Life-Cycle Economies", mimeo, University of Western Ontario
- Greenwood, Jeremy, Zvi Hercowitz and Gregory Huffman (1988), "Investment, Capacity Utilization and the Real Business Cycle", *American Economic Review* 78, 402-417
- Hansen, Gary D. and Edward C. Prescott (1995), "Recursive Methods for Computing Equilibria of Business Cycle Models" in Thomas F. Cooley (ed.), *Frontiers of Business Cycle Research*. Princeton, N.J., Princeton University Press
- James, Steven (1994), "Debt Reduction with Distorting Taxes and Incomplete Ricardianism: A Computable Dynamic General Equilibrium Anal-

- ysis”, in William Robson and William Scarth (eds.), *Deficit Reduction: What Pain, What Gain?*. Toronto, C.D. Howe Institute
- Jones, L.E., R. Manuelli and P.E. Rossi (1993), “Optimal Taxation in Models of Endogenous Growth”, *Journal of Political Economy* 101, 485-517
- Jones, L.E., R. Manuelli and P.E. Rossi (1997), “On the Optimal Taxation of Capital Income”, *Journal of Economic Theory* 73, 93-117
- Judd, Kenneth L. (1985), “Redistributive Taxation in a Simple Perfect Foresight Model”, *Journal of Public Economics* 28, 59-83
- Judd, Kenneth L. (1987), “The Welfare Cost of Factor Taxation in a Perfect Foresight Model”, *Journal of Political Economy* 95, 675-709
- Judd, Kenneth L. (1999), “Optimal Taxation and Spending in General Competitive Growth Models”, *Journal of Public Economics* 71, 1-26
- Klein, Paul and José-Victor Rios-Rull (1999), “Time-Consistent Optimal Fiscal Policy”, mimeo, Institute for International Economic Studies
- Kydland, Finn E. and Edward C. Prescott (1980), “Dynamic Optimal Taxation, Rational Expectations, and Optimal Control”, *Journal of Economic Dynamics and Control* 2, 79-91
- Maskin, E. and J. Tirole (1993), “Markov Perfect Equilibrium”, mimeo, Harvard University
- Mendoza, Enrique G., Assaf Razin and Linda L. Tesar (1994), “Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption”, *Journal of Monetary Economics* 34, 297-323
- Ramsey, Frank (1927), “A Contribution to the Optimal Theory of Taxation”, *Economic Journal* 37, 47-61
- Ramsey, Frank (1928), “A Mathematical Theory of Saving”, *Economic Journal* 38, 543-559
- Rios-Rull, José Victor (1995), “Models with Heterogeneous Agents”, in T.F. Cooley, ed., *Frontiers of Business Cycle Research*. Princeton, Princeton University Press
- Rios-Rull, José Victor (1996), “Life-Cycle Economies and Aggregate Fluctuations”, *Review of Economic Studies* 63, 465-490
- Rios-Rull, José Victor (1999), “Computation of Equilibria in Heterogeneous-Agent Models”, in Ramon Marimon and Andrew Scott, eds., *Computational Methods for the Study of Dynamic Economies*. New York, Oxford University Press
- Scarth, William (1999), “Alternatives for Raising Living Standards”, mimeo, McMaster University
- Stokey, Nancy (1991), “Credible Public Policy”, *Journal of Economic Dynamics and Control* 15, 627-657
- Ventura, Gustavo (1996), “Flat Tax Reform: A Quantitative Assessment”, mimeo, University of Illinois
- Weil, Philippe (1989), “Overlapping Generations of Infinitely-Lived Agents”, *Journal of Public Economics* 38, 183-198

- Yaari, Menahem E. (1965), "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer", *Review of Economic Studies* 32, 137-150
- Zhu, X. (1992), "Optimal Fiscal Policy in a Stochastic Growth Model", *Journal of Economic Theory* 58, 250-289

## Appendix: Pseudo-Algorithm for Numerical Simulations

The numerical algorithm used to calculate the model's equilibrium with Markov-perfect optimal taxation by the government can be summarized as follows.

- First, define some things that are constant across iterations.
  - Define structural parameters and certain constants.
  - Set the convergence criteria.
- Initialize some variables that change across iterations.
  - Initialize steady state values of states and private and public controls.
  - Initialize steady state wage rate and capital rental rate.
  - Initialize coefficients of government feedback rule.
  - Initialize private sector feedback rules.
  - Initialize state transition matrices.
  - Initialize matrices to hold first order conditions of private agents.
  - Initialize matrix to hold value functions for all cohorts.
  - Initialize government's value function.
  - Define one period return function for private agents in a sufficiently general manner that it can be used for all cohorts.
  - Define one period return function for government.
- **DO** until convergence of government feedback rule. (**Beginning of outer DO loop**)
  - **DO** until convergence of private sector feedback rules. (**Beginning of inner DO loop**)
    - \* Update value functions and feedback rules as follows.
    - \* **DO** for cohorts from 1 to I.
      - Calculate quadratic approximation to value function for cohort i.
      - Solve for first order conditions for cohort i.
      - Impose aggregation.

- Insert aggregated first order conditions into correct row of matrix containing first order conditions for all cohorts.
- \* End **DO**.
- \* Solve simultaneous equation system for aggregate feedback rules.
- \* **DO** for cohorts from I to 1.
  - Solve for feedback rules of cohort i.
  - Calculate value function of cohort i given that of i+1.
  - Update matrix containing all value functions.
- \* End **DO**
- \* Check for convergence by comparing new to old aggregate feedback rules.
- \* Update private sector feedback rules.
- \* Update model steady state.
- \* Update steady state real wage and capital rental rate.
- \* Increment loop counter for inner do loop.
- End **DO**. (**End of inner DO loop**)
- Solve for optimal government feedback rule given new private sector feedback rules.
- Check for convergence of government feedback rule. **This checks for overall convergence of the algorithm.**
- Update the government's feedback rule.
- Update model steady state.
- Update steady state real wage and capital rental rate.
- Increment loop counter for outer do loop.
- End **DO**. (**End of outer DO loop**)
- Report results and save some outputs to disk.

Incrementing the inner and outer loop counters allow for stopping the program if an arbitrary maximum number of iterations is exceeded.

Table 1: Parameter Values

Parameter	Value
$\beta$	0.75
$\delta$	0.75
$\alpha$	0.64
$\theta$	3.00
$\sigma$	1.75
$\gamma$	0.66
$\rho_z$	0.40
$g$	0.20
$\bar{z}$	1.00
$\sigma_z$	0.01

Table 2: Steady State Properties

Variable	Value
$\tau_n$	0.165
$\tau_k$	0.203
$B$	-0.008
$y$	0.138
$g$	0.028
$k$	0.029
$c_y$	0.052
$c_o$	0.029
$c/y$	0.591
$i/y$	0.208
$g/y$	0.200
$25 \cdot k/y$	5.220
$b/y$	-0.058
$n$	0.333
$w$	0.265
$R$	1.727
$r$	0.377

Table 3: Variability of Tax Rates

	Model	Klein/Rios-Rull (Commitment)	Klein/Rios-Rull (No Commitment)	Data
<b>Capital Income Tax</b>				
mean	0.203	-0.002	0.650	0.283
s.d.	0.110	0.180	0.110	0.088
c.v.	0.543	85.71	0.170	0.309
<b>Labor Income Tax</b>				
mean	0.165	0.310	0.120	0.248
s.d.	0.030	0.009	0.031	0.024
c.v.	0.181	0.028	0.250	0.097

s.d.: standard deviation

c.v.: coefficient of variation

The second and third columns are from Klein and Rios-Rull (1999, Tables 3 and 4).

The last column is from Chari, Christiano and Kehoe (1995, Table 12.4).

Figure 1:

