Optimal Predictive Tests

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Abstract

This paper develops optimal tests based on sequential predictive moment conditions. We show that an appropriate weighting version of the predictive test achieves the same power as optimal structural change tests proposed by Sowell (1996a,b). Consequently, we can apply directly Sowell's results. Optimal predictive tests for parameter instability and overidentifying restriction stability are proposed. The finite sample properties of LM, Wald, LR-type and predictive tests for parameter instability are studied via a simulation study.

Key Words: Predictive test; optimal test; moment conditions

JEL Classification: C12, C15, C20

1 Introduction

Ghysels and Hall (1990) propose a predictive test for instability of moment conditions with a known breakpoint in the GMM framework when the form of the structural change is unspecified. This test consists of estimating the parameter vector for the first subsample and then evaluating the moment conditions for the second subsample at these parameter values. The test can be applied for the entire set or a subset of moment conditions. Ghysels, Guay and Hall (1997) extend the predictive test to the case where the breakpoint is unknown. In contrast to the case where the breakpoint is known, these authors consider only the test for the entire set of moment conditions.

Sowell (1996a,b) proposes an optimal structural change test based on usual sequential moment conditions. He shows that the test is divided into a test of structural change for the entire vector of parameters and a test of the stability of overidentifying restrictions. An optimal test for parameter instability can be obtained by projecting the entire set of moment conditions onto the subspace identifying the parameter vector. An optimal test for instability of overidentifying restrictions consists of projecting the entire set of moment conditions onto the subspace which is orthogonal to the subspace identifying the parameter vector.

In the first part of this paper, we apply the general approach proposed by Sowell (1996a,b) to construct optimal tests based on sequential predictive moment conditions. While the class of tests considered in this paper is not a priori the same as Sowell, we show that an appropriate weighting of the predictive tests achieves the same asymptotic power as the tests developed by Sowell (1996a,b). The optimal predictive tests are then optimal structural change tests such as defined by Sowell. This result allows us to propose a new optimal predictive test for parameter instability for fixed or unknown breakpoint. This test consists of projecting the entire set of predictive moment conditions onto the subspace identifying the parameters. Previous predictive tests were not directed specifically to detect parameter instability. As shown in Ghysels, Guay and Hall (1997), a predictive test based on the entire set of moment conditions consists of a test for parameter variations and a test of the stability of the overidentifying restrictions. When the structural change occurs by parameters instability, Wald, LM and LR-type tests are more powerful than the predictive tests in Ghysels, Guay and Hall (1997). The predictive test for parameter instability proposed in this paper is asymptotically equivalent to Wald, LM and LR-type tests.

An optimal predictive test for stability of overidentifying restrictions is also proposed. This test is obtained by projecting the predictive test for the entire set of moment conditions onto the subspace which is orthogonal to the subspace identifying the parameter vector. Tables of critical values are provided based on the asymptotic distribution under the null hypothesis of no structural change. A version of the predictive test corresponding to the one developed by Hall and Sen (1999) for overidentifying restriction instability is also presented.

Finally, a simulation study is performed to examine the size and power properties of various parameter stability tests. The study reveals that stability tests based on statistics computed with an unrestricted estimator of the weighting matrix have a important size distortion problem. A version of the predictive test constructed with a restricted estimator of the weighting matrix has better size and power properties than other stability tests.

The paper is organized as follows: Section 2 contains definitions. Sections 3 derives the optimal properties of tests based on predictive moment conditions and proposes predictive tests for parameter instability and overidentifying restriction instability. The design and the results of the simulation study are presented in Section 4. An Appendix contains proofs of the theoretical results of the paper.

2 Definitions of GMM estimators

In this paper, we consider the class of GMM estimators, which subsumes many standard estimators such as quasi-maximum likelihood, certain semi-parametric procedures, as well as least squares and IV procedures.

We present the GMM framework for the full sample and for the case when the sample is divided in two subsamples. The required regularity conditions are presented in Ghysels, Guay and Hall (1997). For the data, we consider triangular arrays of random variables $\{x_{Tt} : 1 \leq t \leq T, T \geq 1\}$ which are defined on a probability space $(\Omega, \Im, \mathbf{P})$. Triangular arrays allow us to examine the local asymptotic power of alternative tests proposed in this paper.

In a general context, the GMM estimator is based on a set of moment conditions:

$$E\left[f(x_{Tt}, \theta_0)\right] = 0,$$

where $f(\cdot)$ is a R^q-valued function of orthogonality conditions and θ_0 is a p-vector of parameters. The restricted GMM estimator for the full sample is defined as follows:

Definition 2.1 *The set of full sample GMM estimators $\{\hat{\theta}_T\}$ is a sequence of random vectors such that:*

$$\hat{\theta}_T = argmin \frac{1}{T}\sum_{t=1}^{T} f(x_{Tt}, \theta)' W_T \frac{1}{T}\sum_{t=1}^{T} f(x_{Tt}, \theta)$$

where W_T is a random nonnegative symmetric matrix.

The optimal weighting matrix W is defined to be the inverse of:

$$S = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} f(x_{Tt}, \theta_0)\right) \in R^{q\times q}.$$

All the following results for the GMM estimation are based on the optimal weighting matrix S^{-1}. This matrix can be consistently estimated using the methods developed by Gallant (1987), Andrews and Monahan (1992) and Newey and West (1994), among others. The asymptotic distribution of $\hat{\theta}_T$ depends also on the following matrix

$$F = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} E\frac{\partial}{\partial\theta'} f(x_{Tt}, \theta_0) \in R^{q\times p}.$$

We now define the unrestricted GMM estimators for the case where the sample is divided into two parts. The first subsample includes the observations $t = 1, \ldots, [T\pi]$ while the second subsample covers $t = [T\pi] + 1, \ldots, T$ where $\pi \in \Pi \subset (0, 1)$. $[T\pi]$ is the time of the potential structural change and $[\cdot]$ is the integer part operator.

Definition 2.2 *The set of first subsample GMM estimators $\{\hat{\theta}_{1T}(\pi)\}$ is a sequence of random vectors such that:*

$$\hat{\theta}_{1T}(\pi) = argmin \frac{1}{[T\pi]}\sum_{t=1}^{[T\pi]} f(x_{Tt}, \theta)' W_1(\pi) \frac{1}{[T\pi]}\sum_{t=1}^{[T\pi]} f(x_{Tt}, \theta).$$

The set of second subsample GMM estimators $\{\hat{\theta}_{2T}(\pi)\}$ is a sequence of random vectors such that:

$$\hat{\theta}_{2T}(\pi) = argmin \frac{1}{T-[T\pi]} \sum_{t=[T\pi]+1}^{T} f(x_{Tt},\theta)' W_2(\pi) \frac{1}{T-[T\pi]} \sum_{t=[T\pi]+1}^{T} f(x_{Tt},\theta)$$

where W_1 and W_2 are random nonnegative symmetric matrices which could depend on π.

The optimal weighting matrices $W_1(\pi)$ and $W_2(\pi)$ are defined to be the inverse of :

$$S_1(\pi) = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{[T\pi]}} \sum_{t=1}^{[T\pi]} f(x_{Tt},\theta_0)\right) \in R^{q\times q},$$

$$S_2(\pi) = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T-[T\pi]}} \sum_{t=[T\pi]+1}^{T} f(x_{Tt},\theta_0)\right) \in R^{q\times q}$$

and we also define the following matrices

$$F_1(\pi) = \lim_{T\to\infty} \frac{1}{[T\pi]} \sum_{t=1}^{[T\pi]} E\frac{\partial}{\partial\theta'} f(x_{Tt},\theta_0) \in R^{q\times p},$$

$$F_2(\pi) = \lim_{T\to\infty} \frac{1}{T-[T\pi]} \sum_{t=[T\pi]+1}^{T} E\frac{\partial}{\partial\theta'} f(x_{Tt},\theta_0) \in R^{q\times p}.$$

To proceed with the presentation of the proposed tests, we define the moment conditions for the first and the second subsample indexed by the potential change point π and by the GMM estimator used to evaluate their closeness to zero. This estimator could be obtained from the full sample ($\hat{\theta}_T$), the first subsample ($\hat{\theta}_{1T}(\pi)$) or the second subsample ($\hat{\theta}_{2T}(\pi)$). Thus,

$$f_1(\pi,\hat{\theta}) = \frac{1}{[T\pi]} \sum_{t=1}^{[T\pi]} f(x_{Tt},\hat{\theta}) \text{ and } f_2(\pi,\hat{\theta}) = \frac{1}{T-[T\pi]} \sum_{t=[T\pi]+1}^{T} f(x_{Tt},\hat{\theta}),$$

where $\hat{\theta}$ equals $\hat{\theta}_T$, $\hat{\theta}_{1T}(\pi)$ or $\hat{\theta}_{2T}(\pi)$ depending on the case under consideration. Estimators of $F_i(\pi)$ denoted $F_i(\pi,\hat{\theta})$ for $i = 1, 2$ and the full sample estimator $F(x_{Tt},\hat{\theta}_T)$ have the same probability limit under the null which is equal to F.

Finally, estimators of $S_1(\pi)$ and $S_2(\pi)$ defined as $S_1(\pi,\hat{\theta})$ and $S_2(\pi,\hat{\theta})$ can be obtained by usual methods (see discussion above). The estimators $S_i(\pi,\hat{\theta})$ and the full sample estimator $S(x_{Tt},\hat{\theta}_T)$ have the same probability limit under the null hypothesis which is equal to S.

3 Optimal Predictive Tests

A predictive test for moment conditions is constructed by evaluating the moment conditions for the second subsample at the estimator obtained with the first subsample which are:

$$f_2(\pi,\hat{\theta}_1) = \frac{1}{T-[T\pi]} \sum_{t=[T\pi]+1}^{T} f(x_{Tt},\hat{\theta}_1). \tag{3.1}$$

In a similar way, a predictive test can also be based on the moment conditions for the first subsample evaluated at the estimator obtained with the second subsample. In this case, the predictive moment conditions are

$$f_1(\pi, \hat{\theta}_2) = \frac{1}{[T\pi]} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_2). \tag{3.2}$$

We consider the following general specification for the sequence of local alternatives for violation of moment conditions:

$$Ef(x_{Tt}, \theta_0) = \frac{h(\eta, s, t/T)}{\sqrt{T}} \tag{3.3}$$

where $h(\eta, s, \pi)$, for $\pi \in [0, 1]$, is a q-dimensional function that can be expressed as uniform limit of step functions, $\eta \in R^i$, $s \in R^j$ such that $0 < s_1 < s_2 < \ldots < s_j < 1$ and θ_0 is in the interior of Θ. The function $h(\cdot)$ allows a wide range of alternative hypotheses (see Sowell (1996b)). In this function, the parameter s denotes the times of the structural change as fraction of the sample size and the vector η defines the local alternatives. To simplify the notation $h(\eta, s, \pi)$ will be noted $h(\pi)$. These general local alternatives are chosen to show that the predictive tests presented in this paper have non trivial power against a large class of alternatives. More importantly, our asymptotic results can be compared to Sowell's results which are based on usual sequential moment conditions. The derivation will first consider the general local alternatives to focus after that to the case with one time structural break.

The following Theorem gives the asymptotic distribution of both statistics (3.1) and (3.2) presented above.

Theorem 3.1 *Under the sequence of local alternatives in 3.3 and Assumptions in Ghysels, Guay and Hall (1997), we have*

$$\begin{aligned} \frac{1}{\sqrt{T}} S_T^{-1/2} \sum_{t=[T\pi]+1}^{T} f(x_{Tt}, \hat{\theta}_{1T}) \quad &\Rightarrow \quad B(1) - B(\pi) + S^{-1/2} \int_{\pi}^{1} h(r)dr - \\ &\quad \frac{(1-\pi)}{\pi} P_F B(\pi) - \frac{(1-\pi)}{\pi} P_F S^{-1/2} \int_0^{\pi} h(r)dr \end{aligned}$$

and

$$\begin{aligned} \frac{1}{\sqrt{T}} S_T^{-1/2} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T}) \quad &\Rightarrow \quad B(\pi) + S^{-1/2} \int_0^{\pi} h(r)dr - \\ &\quad \frac{\pi}{(1-\pi)} P_F \left(B(1) - B(\pi)\right) - \frac{\pi}{(1-\pi)} P_F S^{-1/2} \int_{\pi}^{1} h(r)dr \end{aligned}$$

over Π, *where* $B(\cdot)$ *is a q-vector of independent Brownian motions and* $P_F = S^{-1/2} F \left(F' S^{-1} F\right)^{-1} F' S^{-1/2}$.

Proof: See Appendix[1]

[1] In the Appendix, the proof is done for the case where the moments of the second subsample are evaluated at the estimator obtained with the first subsample. The sketch of the proof is similar for the case where the moments for the first subsample are evaluated at the second subsample estimator.

In the following, we apply the general approach proposed by Sowell (1993, 1996a) to construct optimal tests. Asymptotic optimal tests are a generalization of the Neyman-Pearson approach to the case of two measures. In our case, the two measures correspond to limit processes of the normalized predictive moment conditions under the null and the alternative. The most powerful test is given by the Radon-Nikodym derivative of the probability measure implied by the local alternatives with respect to the probability measure implied by the null. For composite alternatives the functional that implies the optimal test with the greatest weighted average power is the weighted average of the Radon-Nikodym derivative. To obtain optimal tests with predictive moment conditions, we show that an appropriate weighting of the tests based on predictive moment conditions converges to the same limiting processes under the null and the alternative (3.3) as the class of tests developed by Sowell (1996a,b) for the usual moment conditions. The results of Sowell can then be applied directly to this appropriate weighting to obtain optimal predictive tests.

Under the null hypothesis, a version of Corollary 1 of Sowell(1996a) can be easily shown. Thus

$$C\frac{1}{\sqrt{T}}S_T^{-1/2}\sum_{t=[T\pi]+1}^{T} f(x_{Tt},\hat{\theta}_{1T}) \quad\Rightarrow\quad \left[\begin{array}{c} -BB_p(\pi)/\pi \\ B_{q-p}(1)-B_{q-p}(\pi) \end{array}\right].$$

where $BB_p(\pi)$ is a p-dimensional Brownian bridge, $B_{q-p}(\pi)$ is a $q-p$ -dimensional Brownian motion, C is such that $P_F = C'\Lambda C, CC' = I$ and

$$\Lambda = \left[\begin{array}{cc} I_p & 0_{p\times(q-p)} \\ 0_{(q-p)\times p} & 0_{(q-p)\times(q-p)} \end{array}\right].$$

For the first subsample moment conditions evaluated at the estimator obtained with the second subsample, we have

$$C\frac{1}{\sqrt{T}}S_T^{-1/2}\sum_{t=1}^{[T\pi]} f(x_{Tt},\hat{\theta}_{2T}) \quad\Rightarrow\quad \left[\begin{array}{c} BB_p(\pi)/(1-\pi) \\ B_{q-p}(\pi) \end{array}\right]. \tag{3.4}$$

Next, we show that an appropriate weighting of the predictive moment conditions converges to the same limiting stochastic processes under the null and the general alternatives as the usual sequential moment conditions. Let us concentrate on the normalized predictive moment conditions for the first subsample evaluated at the second subsample estimator. Multiplying the left hand side of equation (3.4) by the following matrix

$$I^*(\pi) = \left[\begin{array}{cc} (1-\pi)I_p & 0_{p\times(q-p)} \\ 0_{(q-p)\times p} & I_{(q-p)} \end{array}\right],$$

we obtain the same limiting process under the null as Sowell for the usual moment conditions. Indeed, the limiting stochastic process under the null of the weighted predictive moment conditions is given by:

$$I^*(\pi)C\frac{1}{\sqrt{T}}S_T^{-1/2}\sum_{t=1}^{[T\pi]} f(x_{Tt},\hat{\theta}_{2T}) \quad\Rightarrow\quad Z(\pi) = \left[\begin{array}{c} BB_p(\pi) \\ B_{q-p}(\pi) \end{array}\right] \tag{3.5}$$

with the weighting matrix $I^*(\pi)$. We can show that under the general alternatives (3.3), the limiting stochastic process of the weighted version of (3.4) is given by

$$Z(\pi) = \left[CP_F S^{-1/2}\left(H(\pi) - \pi H(1)\right) + C(I - P_F)S^{-1/2}H(\pi)\right] + \begin{bmatrix} BB_p(\pi) \\ B_{q-p}(\pi) \end{bmatrix}. \tag{3.6}$$

with $H(\pi) = \int_0^\pi h(r)dr$. This limiting stochastic process under the alternative hypotheses is also the same as the one for the usual sequential moment conditions considered by Sowell (1996b). This implies that a statistical test based on an appropriate weighting of the sequential predictive moments achieves the same asymptotic power as optimal tests developed by Sowell.

The limiting processes (3.5) and (3.6) differ only by the drift under the alternative hypotheses. Theorem 3 and Corollary 2 in Sowell (1996a) can be applied to the Radon-Nikodym derivative of the probability measure implied by the null and the alternative to obtain asymptotic most powerful test against the local alternatives (3.3). The greatest weighted average power test will reject the null of moment stability if

$$\int\int \exp\left(\int_0^1 \mu(\eta, s, \pi)' dZ(\pi) - \frac{1}{2}\int_0^1 \mu(\eta, s, \pi)'\mu(\eta, s, \pi)d\pi\right) dR(\eta, s)dJ(s) \geq c_\alpha \tag{3.7}$$

where $R(\eta, s)$ is a weighting distribution for the magnitude of the instability, $J(s)$ is a weighting function for the location of the instability, c_α defined the size alpha of the test and

$$\mu(\eta, s, \pi) = CP_F S^{-1/2}\left(h(\pi) - H(1)\right) + C(I - P_F)S^{-1/2}h(\pi). \tag{3.8}$$

The expressions (3.7) and (3.8) are the same as the ones derived for the usual sequential moment conditions by Sowell (1996a,b). Tests based on sequential predictive moment conditions (with an appropriate weighting) are then an optimal test for structural change as presented by Sowell (1996a,b). It is important to understand that the original predictive moment conditions (3.4) can be recovered by an appropriate choice of $R(\eta, s)$ and $J(s)$.[2]

We now focus on the case of one time structural change alternative with unknown breakpoint. The one time structural change alternative in all moment conditions is characterized by the following function:

$$h(\pi) = U(s - \pi)\eta \tag{3.9}$$

where η is the parameter vector for the magnitude of the structural change. In the case when the moment conditions are violated before the breakpoint s, $U(s - \pi) = 0$ if $s < \pi$ and $U(s - \pi) = 1$ if $s \geq \pi$.

For a choice of $R(\eta, s)$ which is a normal distribution with mean zero and covariance matrix equal to $U(s)^{-1} = \frac{1+c}{c}I(s) - (P_F s(1 - s) + (I - P_F)s)$ and $J(s)$ with density proportional to $\left(\frac{1+c}{c}\right)^{q/2}|U(s)|^{1/2}$, the test statistic can be reduced to [3]

$$\int_0^1 \exp\left(\frac{1}{2}\frac{c}{1+c}Z(s)'Z(s)\right) ds.$$

[2]see Proposition 3.1 for a choice of $R(\eta, s)$ and $J(s)$ corresponding to predictive tests in Ghysels, Guay and Hall (1997).

[3]The choice of $R(\eta, s)$ and $J(s)$ is the same as Sowell (1996b)

The parameter c controls the variance of the normal weighting density. The test statistics corresponding to various c determine power against close or more distant alternatives. In the case of close alternative ($c = 0$), the optimal test statistic takes the average form,

$$\int_0^1 Z(s)'Z(s)ds.$$

For a distant alternative ($c = \infty$), the optimal test statistic takes the exponential form,

$$\int_0^1 \exp\left(\frac{1}{2}Z(s)'Z(s)\right) ds.$$

The supremum form corresponds to the case where $c/(1 + c) \rightarrow \infty$.

The next Proposition shows that the tests presented in Ghysels, Guay and Hall (1997) based on the entire set of predictive moment conditions are optimal tests.

Proposition 3.1 *The predictive statistic tests presented in Ghysels, Guay and Hall (1997) under the alternative (3.9) when the moment conditions are violated break before the breakpoint s corresponds to a choice of $R(\eta, s)$ to be a normal weighting distribution with zero mean and covariance matrix $U(s)$ where $U(s)^{-1} = S^{-1/2}\left(\frac{1+c}{c}\tilde{I}(s) - (P_F s(1 - s) + (I - P_F)s)\right)S^{-1/2}$ with*

$$\tilde{I}(s) = \begin{bmatrix} s(1-s)I_p & 0_{p\times(q-p)} \\ 0_{(q-p)\times p} & sI_{(q-p)} \end{bmatrix}.$$

and $J(s) = |U(s)|^{1/2}|\tilde{I}(s)|^{1/2}\left((1 + c)/c\right)^{q/2}$ with support $s \in \Pi$ a proper subset of $(0, 1)$.[4]

Proof: See Appendix.

By Proposition (3.1), predictive tests developed in Ghysels, Guay and Hall (1997) are optimal structural change tests for an one time structural break when the moment conditions are violated before the breakpoint s.

The predictive statistic corresponding to this choice of $R(\eta, s)$ and $J(s)$ is

$$\left(C\frac{1}{\sqrt{T}}S_T^{-1/2}\sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T})\right)' II^{*-1}(\pi)\left(C\frac{1}{\sqrt{T}}S_T^{-1/2}\sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T})\right)$$

with

$$II^*(\pi) = \begin{bmatrix} \frac{\pi}{(1-\pi)}I_p & 0_{p\times(q-p)} \\ 0_{(q-p)\times p} & \pi I_{(q-p)} \end{bmatrix}.$$

which is asymptotically equivalent to the statistic presented in Ghysels, Guay and Hall (1997) in the case where the moments for the first subsample are evaluated at the estimator obtained with the second subsample. This statistic can be rewritten as:

$$PR_{1T}(\pi) = \left[\frac{1}{\sqrt{[T\pi]}}\sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T}(\pi))\right]' \hat{V}_1^{-1}(\pi)\left[\frac{1}{\sqrt{[T\pi]}}\sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T}(\pi))\right]$$

[4] In fact the proposition holds for any weighting distribution absorbing $|U(s)|^{1/2}|\tilde{I}(s)|^{1/2}\left((1 + c)/c\right)^{q/2}$.

where $\hat{V}_1(\pi)$ is a consistent estimator of

$$V_1(\pi) = S_1(\pi) + \frac{\pi}{(1-\pi)} F_1(\pi) \left[F_2(\pi)' S_2^{-1}(\pi) F_2(\pi)\right]^{-1} F_1(\pi)'.$$

The full sample or the first subsample estimators of V_1 can be used. Statistics for optimal predictive tests for one time jump in all moment conditions are obtained by computing the average, exponential and supremum form as presented before. As presented in Ghysels, Guay and Hall (1997), the predictive statistic for the case where the moments of the second subsample are evaluated at the first subsample estimator is given by:

$$PR_{2T}(\pi) = \left[\frac{1}{\sqrt{T-[T\pi]}} \sum_{t=[T\pi]+1}^{T} f(x_{Tt}, \hat{\theta}_{1T})\right]' \hat{V}_2^{-1}(\pi) \left[\frac{1}{\sqrt{T-[T\pi]}} \sum_{t=[T\pi]+1}^{[T]} f(x_{Tt}, \hat{\theta}_{1T}(\pi))\right]$$

where $\hat{V}_2(\pi)$ is a consistent estimator of

$$V_2(\pi) = S_2(\pi) + \frac{(1-\pi)}{\pi} F_2(\pi) \left[F_1(\pi)' S_1^{-1}(\pi) F_1(\pi)\right]^{-1} F_2(\pi)'. \tag{3.10}$$

Obviously, the full sample or the second subsample estimators of V_2 can be used. The asymptotic distributions of this statistic and asymptotic distributions of the supremum, average and exponential mappings are given in Theorem 2.1 and 2.2 in Ghysels, Guay and Hall (1997). We can show that this statistic is optimal when the moment conditions are violated after the breakpoint.[5]

Predictive statistics presented above test simultaneous identifying and overidentifying restrictions. These tests give no information about the source of instability in contrast to the tests developped by Hall and Sen (1999) and Sowell (1996a). Predictive tests can be constructed to disentangle these two sources of instability. An optimal predictive test for parameter instability is obtained by projecting the predictive moment conditions onto the subspace identifying the parameters. Following Sowell (1996a,b), we consider the following sequence of alternatives for the identifying moment conditions:

$$Ef(x_{Tt}, \theta_0) = P_F \frac{h(\eta, s, t/T)}{\sqrt{T}}. \tag{3.11}$$

Predictive optimal tests for parameter instability are given by the following statistics:

$$PR_{1T}^I(\pi) = \frac{(1-\pi)}{\pi} \left[\frac{1}{\sqrt{T}} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T})\right]' S_T^{-1/2} \hat{P}_F S_T^{-1/2} \left[\frac{1}{\sqrt{T}} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T}(\pi))\right] \tag{3.12}$$

and

$$PR_{2T}^I(\pi) = \frac{\pi}{(1-\pi)} \left[\frac{1}{\sqrt{T}} \sum_{t=[T\pi]+1}^{T} f(x_{Tt}, \hat{\theta}_{1T})\right]' S_T^{-1/2} \hat{P}_F S_T^{-1/2} \left[\frac{1}{\sqrt{T}} \sum_{t=[T\pi]+1}^{[T]} f(x_{Tt}, \hat{\theta}_{1T}(\pi))\right]$$

where $\hat{P}_F$ is a consistent estimator of P_F which is defined in Theorem 3.1.

[5] The proof is similar to the one of Proposition 3.1 and is omitted for brevity.

The $PR^I_{2T}(\pi)$ statistic is asymptotically equivalent to the following statistic:

$$PR^p_{2T}(\pi) = T(1-\pi)\left(\hat{F}_2'\hat{S}_2^{-1}f_2(\pi,\hat{\theta}_{1T}(\pi))\right)'[\hat{V}^p_2(\pi)]^{-1}\hat{F}_2'\hat{S}_2^{-1}f_2(\pi,\hat{\theta}_{1T}(\pi)), \tag{3.13}$$

where $\hat{F}_2 = F_2(\pi,\hat{\theta}_{1T}(\pi))$, $\hat{S}_2 = S_2(\pi,\hat{\theta}_{1T}(\pi))$ and $\hat{V}^p_2(\pi)$ is a consistent estimator of the asymptotic covariance matrix of $(T-[T\pi]))^{1/2}\hat{F}_2'\hat{S}_2^{-1}f_2(\pi,\hat{\theta}_{1T}(\pi))$ which is equal to $F_2(\pi)'S_2^{-1}(\pi)V_2(\pi)S_2^{-1}(\pi)F_2(\pi)$ and $V_2(\pi)$ defined in eq. (3.10). To see the equivalence of the two statistics, the expression $F_2(\pi)'S_2^{-1}(\pi)V_2(\pi)S_2^{-1}(\pi)F_2(\pi)$ is asymptotically equivalent to $\frac{F'S^{-1}F}{\pi}$ and the result follows directly. A similar statistic can be constructed by using the predictive moments for the first subsample evaluated at the second subsample estimator.

The following Theorem gives asymptotic distributions of both statistics presented above

Theorem 3.2 *Under the sequence of local alternative in (3.11) the following processes indexed by π for a given set $\Pi \subset (0,1)$ satisfy:*

$$Pr^I_{iT}(\pi) \Rightarrow Q_p(\pi)$$

where

$$Q_p(\pi) = \frac{BB_r(\pi)'BB_r(\pi)}{\pi(1-\pi)} + \frac{[H(\pi)-\pi H(1)]'S^{-1/2}P_F S^{-1/2}[H(\pi)-\pi H(1)]}{\pi(1-\pi)}$$

and

$$\sup Pr^I_{iT} \Rightarrow \sup_{\pi\in\Pi} Q_p(\pi), \quad avePr^I_{iT} \Rightarrow \int_\Pi Q_p(\pi)d\pi, \quad expPr^I_{iT} \Rightarrow log\left(\int_\Pi \exp[\frac{1}{2}Q_p(\pi)]d\pi\right),$$

for $i = 1, 2$.

These results follow directly from Theorem 3.1 and the application of the continuum mapping theorem (see Pollard (1984)). Theorem 3.2 shows that both statistics Pr^I_{1T} and Pr^I_{1T} are powerful against parameter instability occuring before or after the breakpoint.

Optimal predictive test for stability of overidentifying restrictions is obtained by a projection of the predictive moment conditions onto the subspace orthogonal to identifying restrictions. The overidentifying restrictions are stable if they hold before and after the breakpoint. This is formally stated as $H^O_0(\pi) = H^{O1}_0(\pi) \cap H^{O2}_0(\pi)$ with:

$$\begin{aligned} H^{O1}_0(\pi) : (I_q - P_F)S^{-1/2}E[f_t(x_{Tt},\theta_0)] &= 0 \qquad \forall t = 1,\ldots,[T\pi] \\ H^{O2}_0(\pi) : (I_q - P_F)S^{-1/2}E[f_t(x_{Tt},\theta_0)] &= 0 \qquad \forall t = [T\pi]+1,\ldots,T. \end{aligned}$$

The sequence of local alternatives for overidentifying restriction instability is given by:

$$Ef(x_{Tt},\theta_0) = (I - P_F)\frac{h(\eta,s,t/T)}{\sqrt{T}} \tag{3.14}$$

The predictive statistics to test instability of overidentifying restrictions are:

$$PR^O_{1T}(\pi) = \frac{1}{\pi}\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{[T\pi]} f(x_{Tt},\hat{\theta}_{2T})\right]' S_T^{-1/2}\left(I-\hat{P}_F\right)S_T^{-1/2}\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{[T\pi]} f(x_{Tt},\hat{\theta}_{2T}(\pi))\right]$$

and

$$PR_{2T}^{O}(\pi)=\frac{1}{(1-\pi)}\left[\frac{1}{\sqrt{T}}\sum_{t=[T\pi]+1}^{T}f(x_{Tt},\hat{\theta}_{1T})\right]'S_T^{-1/2}\left(I-\hat{P}_F\right)S_T^{-1/2}\left[\frac{1}{\sqrt{T}}\sum_{t=[T\pi]+1}^{[T]}f(x_{Tt},\hat{\theta}_{1T}(\pi))\right].$$

The $PR_{2T}^{O}(\pi)$ is aymptotically equivalent to the following statistic:

$$\begin{aligned}PR_{2T}^{*,0}(\pi) &= T(1-\pi)\left((I-\hat{P}_{F_2})(\pi)'\hat{S}_2^{-1/2}(\pi)f_2(\pi,\hat{\theta}_{1T}(\pi))\right)'\hat{V}_{I-P_{F_2}}^{-1}(\pi)\ \times\\ &\quad \left((I-\hat{P}_{F_2}(\pi))\hat{S}_2^{-1/2}(\pi)f_2(\pi,\hat{\theta}_{1T}(\pi))\right),\end{aligned}$$

where $\hat{S}_2(\pi)=\hat{S}_2(\pi,\hat{\theta}_{1T}(\pi))$ and $\hat{P}_{F_2}(\pi)$ is a consistent estimator of $S_2^{-1/2}(\pi)F_2(\pi)\left(F_2(\pi)'S_2^{-1}(\pi)F_2(\pi)\right)^{-1}\times F_2(\pi)'S_2^{-1/2}(\pi)$. The matrix $\hat{V}_{I-P_{F_2}}(\pi)$ is a consistent estimator of the asymptotic covariance matrix of $(T(1-\pi))^{1/2}(I-\hat{P}_{F_2}(\pi))\hat{S}_2^{-1/2}f_2(\pi,\hat{\theta}_{1T}(\pi))$ which is equal to

$$(I-P_{F_2}(\pi))S_2^{-1/2}(\pi)V_2(\pi)S_2^{-1/2}(\pi)(I-P_{F_2}(\pi)).$$

This expression is equal asymptotically to $(I-P_F)$ and the asymptotic equivalence for $PR_{2T}^{O}(\pi)$ and $PR_{2T}^{*,0}(\pi)$ follows directly. A similar statistic can be constructed by using the predictive moment conditions for the first subsample evaluated at the second subsample estimator.

Theorem 3.3 *Under the sequence of local alternatives in 3.14, the following processes indexed by π for a given set $\Pi\subset(0,1)$ satisfies:*

$$Pr_{1T}^{O}(\pi)\Rightarrow Q_{1,q-p}^{*}(\pi)$$

$$Pr_{2T}^{O}(\pi)\Rightarrow Q_{2,q-p}^{*}(\pi)$$

where

$$Q_{1,q-p}^{*}(\pi)=\frac{B_{q-p}(\pi)'B_{q-p}(\pi)}{\pi}+\frac{H(\pi)S^{-1/2}(I-P_F)S^{-1/2}H(\pi)}{\pi}$$

$$Q_{2,q-p}^{*}(\pi)=\frac{\left[B_{q-p}(1)-B_{q-p}(\pi)\right]'\left[B_{q-p}(1)-B_{q-p}(\pi)\right]}{1-\pi}+\frac{\left[H(1)-H(\pi)\right]'S^{-1/2}(I-P_F)S^{-1/2}\left[H(1)-H(\pi)\right]}{1-\pi}$$

and

$$\sup Pr_{iT}^{O}\Rightarrow\sup_{\pi\in\Pi}Q_{i,q-p}^{*}(\pi),\quad avePr_{iT}^{O}\Rightarrow\int_{\Pi}Q_{i,q-p}^{*}(\pi)d\pi,\quad expPr_{iT}^{O}\Rightarrow log\left(\int_{\Pi}\exp[\frac{1}{2}Q_{i,q-p}^{*}(\pi)]d\pi\right),$$

where $i=1,2$.

These results follow directly from Theorem 3.1 and the application of the continuum mapping theorem (see Pollard (1984)). The critical values of continuous mappings supremum, average and exponential are presented in Tables 1 to 3 for the intervals indexed by π_0 and $(1-\pi_0)$. Theorem 3.3 shows that the test statistic $Pr_{1T}^{O}(\pi)$ is an optimal test against violation of overidentifying restrictions before the breakpoint and $Pr_{2T}^{O}(\pi)$ is an optimal test against violation of overidentifying restrictions after the breakpoint.[6]

[6]see Hall and Sen (1996) for an interpretation of optimality before and after the breakpoint.

A statistic in the same spirit of Hall and Sen (1999) to test stability of overidentifying restrictions can be constructed. This statistic is

$$PR_T^O(\pi) = PR_{1T}^O(\pi) + PR_{2T}^O(\pi).$$

As pointed by Hall and Sen, this statistic is powerful against violation of $H_0^{01}(\pi)$ and $H_0^{02}(\pi)$. The critical values of the usual mappings of this statistic are given in Table 1 of Hall and Sen (1999).

4 Simulation study

We conduct a Monte carlo study to appraise the finite sample properties of structural change tests for parameter instability. We consider the following AR(1) model:

$$y_t = \rho y_{t-1} + \epsilon_t$$

where ϵ_t is i.i.d. $N(0,1)$. The parameter ρ is estimated by the two-steps GMM estimator with y_{t-1} and y_{t-2} as instruments. The moment conditions are then:

$$f(x_{Tt}, \theta) = (y_{t-1}, y_{t-2})'(y_t - \rho y_{t-1}).$$

The parameter stability for ρ is assessed for Wald, LM, LR-type tests and three versions of the predictive test. The first version (PR1) uses the restricted estimator of P_F and corresponds to the statistic (3.12). The second version (PR2) is based on statistic (3.13) and is contructed with the unrestricted estimator of $V_2^p(\pi)$. The last version (PR3) is the same as the second except that the unrestristed estimators of the weighting matrices $S_1(\pi)$ and $S_2(\pi)$ are replaced by the restricted estimator S_T. The heteroscedasticity and autocorrelation consistent covariance matrix estimator used the sample moments in mean deviation as suggested by Hall (2000). The estimator of the covariance matrix is the one proposed by Newey and West (1994) with automatic lag selection. The sample sizes are $T = 100$, $T = 200$ and $T = 500$. Under the alternative breaks occur at $s = .25, .50$ and $.75$. The results are reported for the average, the exponential and the supremum mappings. The possible breakpoint is included between 15% and 85% of the sample. Under the null, the parameter ρ is fixed to 0, .5 and .9. Under the alternative we have the following design:

$$\rho_1 = 0 \text{ and } \rho_2 = .1, .25, .50,$$
$$\rho_1 = .5 \text{ and } \rho_2 = 0, .25, .60,$$
$$\rho_1 = .9 \text{ and } \rho_2 = .4, .65, .80,$$

where ρ_1 is the value of the parameter for the first part of the sample and ρ_2 is the value for the second part. The values for the alternative are chosen such as the distance from the null δ is equal to .1, .25 and .50.

The Monte Carlo study is based on 1000 replications. For each test, the standard error of the percentage of rejections correponds to $(R(1-R))/1000)^{1/2}$ where R is the percentage of rejections.

In table 4, we report the level properties for the average, exponential and supremum mappings of the tests. First we note that tests calculated with an unrestricted estimator of the weighting matrix have serious size distortions. For example, the supremum test based on the LR-type statistic rejects up to 40 % for a sample of 100 observations. The size problem is more serious for the LR and Wald statistic tests than the $PR2$ version of the predictive statistic test. It is also important to note that for each case, LR-type statistic test rejects more often than Wald statistic test. These results seem to contradict the results obtained in Ghysels and al. (1997) for the same design under the null. In Ghysels and al. (1997), the weighting matrix is restricted to correspond to the fact that the variance estimator of the residual vector is given by $\hat{\sigma}^2 I_T$ where $\hat{\sigma}^2$ is the usual estimator of $\sigma^2 = E(\epsilon^2)$. In our Monte Carlo experiments, the estimation of the weighting matrix is not restricted and the choice of the bandwith order is selected by the automatic lag selection procedure proposed by Newey and West (1994).[7] In practice, the weighting matrix is not restricted and the order of autocorrelation for the moments conditions is unknown. The strategy adopted in these simulations is then more realistic. For sake of comparaison, Table 5 reports the level properties when we impose that the variance of the error terms is $\hat{\sigma}^2 I_T$ as in Ghysels, Guay and Hall (1997). The size is now closer to the true one for all statistics. The LM statistic is slightly more conservative than the LR and Wald statistics in small samples while the statistics based on predictive moment conditions are close to LR and Wald statistics. Because, as mentioned above, the order of lags to include in the bandwith is unknown and the estimation of the weighting matrix is not restricted, we consider, in the rest of simulations, the automatic lag procedure proposed by Newey and West (1994).

The parameter stability tests based on LM and $PR3$ statistics have rejection frequencies close to the nominal size. In particular for ρ equal to 0 and .5, the PR statistic test has a rejection frequencies very close to 5 % while LM statistic test is too conservative. For $\rho = .9$, the LM test is closer to the nominal size than the $PR2$ statistic test which is too conservative. Stability test based on $PR1$ statistic has a proper rejection frequencies for low and moderate persistences but overrejects for process with high persistence ($\rho = .9$).

In Tables 6, 7 and 8, we examine the power properties of the tests when the breakpoint of the $AR(1)$ model occurs at fraction $s = .25, .50$ and $.75$ of the sample. Table 6 contains results for the average mapping, Table 7 for the exponential mapping and Table 8 for the supremum mapping.

For almost all cases the power properties are symmetric with regard to the location of the breakpoint. As expected in the case of a small structural change ($\delta = .10$) the average form dominates exponential and supremum forms. For the larger structural change ($\delta = .50$) the exponential form dominates average form in medium and large samples. When $T = 100$ the results are mixed. Finally, the supremum form is less powerful than the average and exponential forms in almost all cases.

Because of the importance of the size distortions occurring with the Wald, LR, $PR1$ and $PR2$ statistic

[7]Automatic lags selection procedure based on a different kernel proposed by Andrews and Monahan (1992) yields similar results.

tests, we ignore these tests. A size corrected version of these tests could be studied. However, the usual practitioner does not use size corrected critical values. The third version of the predictive tests seems to be more powerful than the LM tests for an $AR(1)$ with low and moderate persistences. For $\rho = .9$, the stability test based on LM statistic is more powerful in the cases where the distance from the null is low or moderate. When the distance from the null is equal to .5 the third version of the predictive test is more powerful than is competitor.

This simulation study showed that test statistics constructed with an unrestricted estimator of the weighting matrix must be clearly avoided. Indeed, the size distortion occuring with those tests could be important even in large sample. A version of predictive test computed with the restricted estimator of the weighting matrix has better size and power properties in the majority of the cases compare to the LM test.

5 Conclusion

In this paper we show that tests based on predictive moment conditions achieves the same asymptotic power as optimal structural change tests proposed by Sowell (1996a, 1996b). Optimal predictive tests are then optimal structural change tests. Moreover, we introduce optimal predictive tests for instability of parameter and overidentifying restrictions. A simulation study reveals that stability tests based on statistics computed with an unrestricted estimator of the weighting matrix have important size distortion problem. A version of the predictive test constructed with a restricted estimator of the weighting matrix has better size and power properties than other stability tests. A more elaborated simulation study is left for future work. In particular, the properties of stability tests in nonlinear models have to be investigated. The performance of stability tests for overidentifying restrictions should be also investigated.

A Appendix

Proof of Theorem 3.1: Under Assumptions given in Ghysels, Guay and Hall (1997) and the alternative (3.3), we can show that

$$\frac{1}{\sqrt{T}} S_T^{-1/2} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \theta_0) \Rightarrow B(\pi) + S^{-1/2} \int_0^{\pi} h(r)dr \tag{A.1}$$

and

$$\frac{1}{\sqrt{T}} S_T^{-1/2} \sum_{t=[T\pi]+1}^{T} f(x_{Tt}, \theta_0) \Rightarrow B(1) - B(\pi) + S^{-1/2} \int_{\pi}^{1} h(r)dr. \tag{A.2}$$

Let us consider an expansion of the moment conditions for the second sample evaluated with its respective estimator

$$\frac{1}{T-[T\pi]} \sum_{t=[t\pi]+1}^{T} f(x_{Tt}, \hat{\theta}_{2T}) = \frac{1}{T-[T\pi]} \sum_{t=[t\pi]+1}^{T} f(x_{Tt}, \theta_0) + F_2(\pi, \bar{\theta}) \left(\hat{\theta}_{2T} - \theta_0\right).$$

where $\bar{\theta} = [\bar{\theta}^1 \dots \bar{\theta}^p]$ and $\bar{\theta}^k = \lambda^{(k)}\theta_0^{(k)} + (1-\lambda^{(k)})\hat{\theta}_{2T}^{(k)}$ for some $0 \leq \lambda^{(k)} \leq 1$ and $k = 1, \dots, p$. Multiplying both sides by: $F_2(\pi, \hat{\theta}_{2T})' S_2(\pi, \hat{\theta}_{2T})^{-1}$ we obtain

$$\left(\hat{\theta}_{2T} - \theta_0\right) = -\left[F_2(\pi, \hat{\theta}_{2T})' S_2(\pi, \hat{\theta}_{2T})^{-1} F_2(\pi, \bar{\theta})\right]^{-1} F_2(\pi, \hat{\theta}_{2T})' S_2(\pi, \hat{\theta}_{2T})^{-1} \frac{1}{[T(1-\pi)]} \sum_{t=[t\pi]+1}^{T} f(x_{Tt}, \theta_0) \tag{A.3}$$

since $F_2(\pi, \hat{\theta}_{2T})' S_2(\pi, \hat{\theta}_{2T})^{-1} \frac{1}{T-[T\pi]} \sum_{t=[t\pi]+1}^{T} f(x_{Tt}, \hat{\theta}_{2T}) = o_p(1)$.

Now we expand the moment conditions for the first subsample evaluated at the second sample estimator

$$\frac{1}{[T\pi]} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T}) = \frac{1}{[T\pi]} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \theta_0) + F_1(\pi, \bar{\theta}) \left(\hat{\theta}_{2T} - \theta_0\right). \tag{A.4}$$

where $\bar{\theta}$ is defined above except that $\hat{\theta}_{1T}^{(k)}$ replaces $\hat{\theta}_{2T}^{(k)}$. We substitue (A.3) into (A.4)

$$\begin{aligned} \frac{1}{[T\pi]} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T}) &= \frac{1}{[T\pi]} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \theta_0) - F_1(\pi, \bar{\theta}) \left[F_2(\pi, \hat{\theta}_{2T})' S_2(\pi, \hat{\theta}_{2T})^{-1} F_2(\pi, \bar{\theta})\right]^{-1} \times \\ & F_2(\pi, \hat{\theta}_{2T})' S_2(\pi, \hat{\theta}_{2T})^{-1} \frac{1}{[T(1-\pi)]} \sum_{t=[t\pi]+1}^{T} f(x_{Tt}, \theta_0) \end{aligned}$$

By (A.1), (A.2), Assumptions in Ghysels, Guay and Hall (1997) and the consistency of $\bar{\theta}$, we obtain

$$\begin{aligned} \frac{1}{\sqrt{T}} S_T^{-1/2} \sum_{t=1}^{[T\pi]} f(x_{Tt}, \hat{\theta}_{2T}) \quad &\Rightarrow \quad B(\pi) + S^{-1/2} \int_0^{\pi} h(r)dr - \\ & \frac{\pi}{(1-\pi)} S^{-1/2} F(F'S^{-1}F)^{-1} F' S^{-1/2} \left[B(1) - B(\pi) + S^{-1/2} \int_{\pi}^{1} h(r)dr\right]. \end{aligned}$$

Proof of Proposition 3.1:

The one time structural change alternative is characterized by the following function

$$h(\pi) = U(s-\pi)\eta$$

where η is the parameter vector for the magnitude of the structural change. In the case when the moment conditions are violated before the breakpoint s, $U(s-\pi) = 0$ if $s < \pi$ and $U(s-\pi) = 1$ if $s \geq \pi$. In this case, the drift becomes

$$\begin{aligned} \mu(\eta, s, \pi) \quad &= \quad C P_F S^{-1/2}(U(s-\pi) - s)\eta + C(I - P_F) S^{-1/2} U(s-\pi)\eta \\ & \Lambda C S^{-1/2}(U(s-\pi) - s)\eta + (I - \Lambda) C S^{-1/2} U(s-\pi)\eta \end{aligned}$$

by $P_F = C'\Lambda C$. Thus for $0 \leq \pi \leq s$, $\mu(\eta, s, \pi) = (1-s)\Lambda C S^{-1/2}\eta + (I-\Lambda) C S^{-1/2}\eta$ and for $s \leq \pi \leq 1$, $\mu(\eta, s, \pi) = -s\Lambda C S^{-1/2}\eta$. The greatest weighted average power test is:

$$\int\int \exp\left(\int_0^1 \mu(\eta, s, \pi)' dZ(\pi) - \frac{1}{2}\int_0^1 \mu(\eta, s, \pi)'\mu(\eta, s, \pi) d\pi\right) dR(\eta, s) dJ(s) \geq c_\alpha \tag{A.5}$$

which gives

$$\begin{aligned} &\int\int \exp\left\{\eta' S^{-1/2} C' \Lambda Z(s) - s\eta' S^{-1/2} C' \Lambda Z(1) + \eta' S^{-1/2} C'(I - \Lambda) Z(s)\right\} \times \\ \exp&\left\{-\frac{1}{2} s(1-s)\eta' S^{-1/2} C' \Lambda C S^{-1/2}\eta - \frac{1}{2} s\eta' S^{-1/2}(I - C'\Lambda C) S^{-1/2}\eta\right\} dR(\eta, s) dJ(s). \end{aligned}$$

by $CC' = I$. The term $S^{-1/2}C'\Lambda Z(1) = 0$ since $\Lambda I^*(\pi) = 0$ for $\pi = 1$ (see 3.5). The above expression can then be rewritten as:

$$\int\int \exp\left\{\eta' S^{-1/2} C' Z(s) - \frac{1}{2}\eta' S^{-1/2}\left(s(1-s) C'\Lambda C + s(I - C'\Lambda C)\right) S^{-1/2}\eta\right\} dR(\eta, s) dJ(s).$$

For a choice of $R(\eta, s)$ to be a normal weighting distribution with zero mean and covariance matrix $U(s)$ where $U(s)^{-1} = S^{-1/2}\left(\frac{1+c}{c}\tilde{I}(s) - (s(1-s)P_F + s(I - P_F))\right)S^{-1/2}$ with

$$\tilde{I}(s) = \begin{bmatrix} s(1-s)I_p & 0_{p\times(q-p)} \\ 0_{(q-p)\times p} & sI_{(q-p)} \end{bmatrix},$$

the expression above yields

$$\int (2\pi)^{-q/2}|U(s)|^{-1/2}\left[\int \exp\left\{\eta' S^{-1/2}C'Z(s) - \frac{1}{2}\eta' S^{-1/2}\left(s(1-s)C'\Lambda C + s(I - C'\Lambda C)\right)S^{-1/2}\eta\right\}d\eta\right] \times$$
$$\left[\int \exp\left\{-\frac{1}{2}\eta' S^{-1/2}\left(\frac{1+c}{c}\tilde{I}(s) - s(1-s)P_F + s(I-P_F)\right)S^{-1/2}\eta\right\}d\eta\right]dJ(s)$$

which is equal to

$$\int (2\pi)^{-q/2}|U(s)|^{-1/2}\left[\int \exp\left\{\eta' S^{-1/2}C'Z(s) - \frac{1}{2}\eta' S^{-1/2}\left(\frac{1+c}{c}\tilde{I}(s)\right)S^{-1/2}\eta\right\}d\eta\right]dJ(s).$$

since $P_F = C'\Lambda C$. After some manipulations, the expression above becomes

$$\int (2\pi)^{-q/2}|U(s)|^{-1/2}\exp\left\{\frac{1}{2}\frac{c}{1+c}Z(s)'C\tilde{I}(s)^{-1}C'Z(s)\right\} \times$$
$$\left[\int \exp\left\{-\frac{1}{2}\left(S^{-1/2}\eta - \frac{c}{1+c}\tilde{I}(s)^{-1}C'Z(s)\right)'\left(\frac{1+c}{c}\tilde{I}(s)\right)\left(S^{-1/2}\eta - \frac{c}{1+c}\tilde{I}(s)^{-1}C'Z(s)\right)\right\}d\eta\right]dJ(s).$$

Since

$$(2\pi)^{-q/2}\left(\frac{1+c}{c}\right)^{q/2}|\tilde{I}(s)|^{1/2}\int \exp\left\{-\frac{1}{2}\left(S^{-1/2}\eta - \frac{c}{1+c}\tilde{I}(s)^{-1}C'Z(s)\right)'\left(\frac{1+c}{c}\tilde{I}(s)\right)\left(S^{-1/2}\eta - \frac{c}{1+c}\tilde{I}(s)^{-1}C'Z(s)\right)\right\}d\eta = 1,$$

we obtain

$$\int |U(s)|^{-1/2}\left(\frac{1+c}{c}\right)^{-q/2}|\tilde{I}(s)|^{-1/2}\exp\left\{\frac{1}{2}\frac{c}{1+c}Z(s)'C\tilde{I}(s)^{-1}C'Z(s)\right\}dJ(s).$$

For any weighting distribution absorbing $|U(s)|^{1/2}|\tilde{I}(s)|^{1/2}\left((1+c)/c\right)^{q/2}$, and by $CC' = I$ the result follows.

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Table 1: Asymptotic critical values for supremum mapping

π_0	$q-p=1$			$q-p=2$			$q-p=3$			$q-p=4$			$q-p=5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
.490	3.10	4.22	6.96	5.15	6.57	9.95	6.93	8.49	12.05	8.58	10.43	14.25	10.11	12.06	16.26
.480	3.31	4.50	7.26	5.42	6.82	10.37	7.26	8.77	12.57	8.98	10.83	14.59	10.48	12.38	16.75
.470	3.46	4.66	7.51	5.65	7.11	10.73	7.51	9.09	12.72	9.25	11.15	15.02	10.77	12.79	17.14
.460	3.62	4.81	7.67	5.85	7.34	10.88	7.67	9.26	13.03	9.49	11.37	15.24	11.01	12.99	17.53
.450	3.74	4.94	7.82	6.07	7.51	11.04	7.81	9.46	13.20	9.69	11.59	15.33	11.22	13.17	17.66
.440	3.82	5.09	7.91	6.18	6.68	11.30	7.99	9.59	13.31	9.85	11.79	15.51	11.46	13.42	17.93
.420	4.08	5.36	8.28	6.43	7.98	11.48	8.27	9.93	13.55	10.15	12.21	15.82	11.77	13.80	18.17
.400	4.27	5.54	8.52	6.62	8.21	11.88	8.60	10.27	13.87	10.44	12.54	16.11	12.05	14.12	18.67
.380	4.43	5.68	8.88	6.76	8.44	12.21	8.83	10.56	14.11	10.69	12.83	16.49	12.29	14.38	18.91
.360	4.56	5.85	9.04	6.97	8.60	12.50	9.04	10.76	14.49	10.93	13.10	16.93	12.51	14.56	19.01
.350	4.63	5.96	9.19	7.07	8.70	12.54	9.15	10.80	14.53	11.04	13.16	17.22	12.59	14.73	19.08
.340	4.70	6.02	9.27	7.13	8.83	12.59	9.22	10.88	14.61	11.20	13.22	17.34	12.72	14.90	19.22
.320	4.86	6.18	9.43	7.33	8.95	12.76	9.39	11.03	14.94	11.38	13.43	17.56	12.93	15.14	19.54
.300	5.01	6.32	9.62	7.49	9.14	13.05	9.58	11.27	15.05	11.60	13.63	17.76	13.11	15.26	19.73
.280	5.17	6.55	9.78	7.68	9.25	13.23	9.70	11.46	15.07	11.78	13.79	17.90	13.35	15.48	19.92
.260	5.27	6.69	9.86	7.82	9.40	13.49	9.83	11.65	15.26	11.93	14.00	18.11	13.58	15.72	20.13
.250	5.33	6.78	10.00	7.89	9.47	13.57	9.94	11.71	15.41	12.04	14.07	18.19	13.66	15.83	20.15
.240	5.40	6.81	10.10	7.96	9.53	13.64	10.01	11.79	15.55	12.12	14.14	18.24	13.76	15.90	20.27
.220	5.53	6.92	10.28	8.05	9.72	13.74	10.18	11.90	15.61	12.30	14.25	18.41	13.97	16.10	20.48
.200	5.65	7.08	10.47	8.22	9.85	13.87	10.32	12.20	15.71	12.47	14.44	18.73	14.21	16.40	20.71
.180	5.79	7.14	10.62	8.35	9.99	14.02	10.46	12.37	16.05	12.64	14.58	18.96	14.37	16.57	20.76
.160	5.94	7.30	10.72	8.50	10.13	14.12	10.62	12.50	16.08	12.82	14.78	19.18	14.54	16.75	20.97
.150	6.02	7.37	10.79	8.59	10.24	14.16	10.71	12.53	16.12	12.90	14.87	19.35	14.64	16.80	21.09
.140	6.06	7.45	10.81	8.64	10.35	14.28	10.79	12.59	16.19	16.03	14.95	19.44	14.73	16.87	21.18
.120	6.22	7.62	11.00	8.79	10.53	14.53	10.88	12.72	16.23	13.19	15.18	19.64	14.93	17.06	21.45
.100	6.35	7.77	11.38	8.95	10.70	14.61	11.07	12.85	16.44	13.37	15.38	19.92	15.14	17.22	21.70
.080	6.55	7.95	11.78	9.17	10.94	14.70	11.32	13.05	16.69	13.58	15.55	20.08	15.37	17.34	21.89
.060	6.71	8.14	11.88	9.38	11.13	14.95	11.57	13.34	17.10	13.82	15.82	20.36	15.64	17.59	22.02
.050	6.79	8.24	12.01	9.52	11.30	15.01	11.72	13.49	17.24	13.97	15.95	20.42	15.79	17.82	22.12
.040	6.88	8.45	12.06	9.67	11.42	15.31	11.88	13.66	17.41	14.09	16.06	20.60	16.00	17.97	22.28
.030	6.99	8.52	12.15	9.84	11.61	15.67	12.05	13.81	17.62	14.24	16.22	20.69	16.22	18.26	22.48
.020	7.13	8.64	12.22	10.00	11.75	15.98	12.28	13.94	17.70	14.45	16.45	20.82	16.45	18.43	22.95

π_0	$q-p=6$			$q-p=7$			$q-p=8$			$q-p=9$			$q-p=10$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
.490	11.64	13.51	18.12	13.04	15.04	19.59	14.42	16.63	21.86	15.66	18.06	22.98	17.07	19.39	24.46
.480	12.03	13.98	18.53	13.45	15.56	19.93	14.95	17.08	22.38	16.20	18.58	23.42	17.54	20.03	25.12
.470	12.29	14.39	18.76	13.78	15.84	20.63	15.24	17.52	22.87	16.60	19.08	23.99	17.90	20.44	25.52
.460	12.57	14.69	19.09	14.04	16.07	20.84	15.50	17.79	23.16	16.87	19.45	24.27	18.21	20.78	25.71
.450	12.89	14.96	19.43	14.30	16.37	20.98	15.78	18.18	23.48	17.10	19.67	24.56	18.47	21.10	26.18
.440	13.05	15.15	19.74	14.51	16.55	21.15	16.05	18.38	23.58	17.35	19.94	24.88	18.66	21.32	26.40
.420	13.42	15.51	20.18	14.85	16.94	21.57	16.51	18.90	24.02	17.78	20.32	25.29	19.16	21.70	26.91
.400	13.71	15.79	20.45	15.23	17.29	21.86	16.85	19.23	24.47	18.18	20.58	25.70	19.46	22.00	27.29
.380	13.95	16.05	20.75	15.44	17.67	22.31	17.17	19.62	24.62	18.53	20.94	25.75	19.78	22.32	27.67
.360	14.23	16.33	21.17	15.70	17.96	22.80	17.45	19.79	24.78	18.87	21.28	26.16	20.17	22.63	27.92
.350	14.35	16.50	21.19	15.81	18.10	22.86	17.56	19.88	24.87	19.01	21.51	26.20	20.30	22.75	28.10
.340	14.42	16.58	21.30	15.94	18.18	22.99	17.68	19.97	24.96	19.10	21.58	26.37	20.46	22.91	28.29
.320	14.68	16.79	21.48	16.21	18.37	23.25	17.86	20.18	25.20	19.41	21.78	26.57	20.70	23.19	28.45
.300	14.87	17.03	21.71	16.46	18.62	23.48	18.10	20.34	25.42	19.57	21.96	26.73	20.97	23.36	28.77
.280	15.11	17.26	21.85	16.60	18.83	23.63	18.28	20.63	25.75	19.82	22.10	27.07	21.23	23.57	28.85
.260	15.37	17.52	21.99	16.83	19.09	23.86	18.51	20.87	26.01	20.06	22.32	27.27	21.41	23.82	19.09
.250	15.45	17.59	22.26	16.94	19.22	23.89	18.59	20.96	26.15	20.14	22.45	27.33	21.50	23.87	29.25
.240	15.50	17.71	22.31	17.01	19.28	24.01	18.64	21.13	26.23	20.21	22.54	27.41	21.60	23.98	29.28
.220	15.68	17.87	22.64	17.22	19.46	24.19	18.87	21.29	26.51	20.38	22.65	27.60	21.84	24.19	29.47
.200	15.89	17.99	22.70	17.40	19.66	24.27	19.07	21.57	26.68	20.58	22.86	27.76	22.06	24.46	29.68
.180	16.13	18.24	22.88	17.59	19.83	24.32	19.35	21.80	26.69	20.78	23.01	27.97	22.26	24.70	29.88
.160	16.30	18.45	23.14	17.86	20.05	24.58	59.57	21.98	26.95	21.07	23.31	28.04	22.48	24.90	30.22
.150	16.39	18.53	23.21	17.92	20.10	24.62	19.72	22.18	26.98	21.17	23.45	28.29	22.62	25.00	30.25
.140	16.52	18.67	23.38	18.02	20.28	24.75	19.80	22.30	27.01	21.25	23.53	28.33	22.74	25.16	30.36
.120	16.67	18.86	23.68	18.24	20.51	24.91	20.03	22.45	27.22	21.54	23.82	28.61	23.00	25.50	30.51
.100	16.91	18.97	23.82	18.46	20.74	25.28	20.24	22.63	27.51	21.78	24.15	28.85	23.25	25.75	30.81
.080	17.18	19.24	24.26	18.62	21.02	25.73	20.52	23.01	27.83	21.99	24.38	29.10	23.60	26.08	31.02
.060	17.39	19.44	24.53	18.96	21.30	26.06	20.77	23.20	28.04	22.31	24.61	29.53	23.91	26.34	31.54
.050	17.57	19.66	24.59	19.18	21.54	26.38	21.00	23.70	28.06	22.45	24.86	29.76	24.06	26.39	31.60
.040	17.72	19.83	24.65	.19.39	21.70	26.77	21.19	23.55	28.18	22.68	25.08	29.84	24.33	26.71	37.77
.030	17.87	20.03	24.93	19.57	21.85	26.83	21.40	23.77	28.30	22.85	25.28	29.93	24.51	26.87	31.91
.020	18.10	20.24	25.10	19.87	22.12	27.20	21.70	23.94	28.66	23.20	25.56	30.24	24.72	27.12	32.45

Table 2 Asymptotic critical values for exponential mapping

π_0	$q-p=1$			$q-p=2$			$q-p=3$			$q-p=4$			$q-p=5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
.490	1.34	1.87	3.19	2.30	2.96	4.59	3.14	3.87	5.63	3.93	4.83	6.67	4.67	5.58	7.64
.480	1.33	1.87	3.15	2.32	2.97	4.60	3.15	3.87	5.62	3.96	4.86	6.67	4.68	5.59	7.68
.470	1.34	1.86	3.16	2.31	2.98	4.62	3.16	3.89	5.66	3.98	4.86	6.69	4.68	5.61	7.69
.460	1.35	1.87	3.12	2.34	2.98	4.65	3.18	3.92	5.69	3.99	4.86	6.67	4.69	5.63	7.72
.450	1.34	1.87	3.13	2.34	3.00	4.63	3.17	3.92	5.64	4.01	4.89	6.64	4.72	5.65	7.75
.440	1.35	1.87	3.14	2.36	3.01	4.64	3.17	3.92	5.64	4.03	4.91	6.66	4.73	5.67	7.77
.420	1.35	1.88	3.18	2.35	3.01	4.68	3.18	3.92	5.60	4.05	4.93	6.71	4.76	5.70	7.73
.400	1.37	1.88	3.19	2.35	3.04	4.70	3.21	3.94	5.64	4.05	4.95	6.68	4.77	5.72	7.72
.380	1.37	1.89	3.21	2.36	3.05	4.67	3.22	3.97	5.63	4.06	4.98	6.68	4.79	5.74	7.77
.360	1.37	1.90	3.20	2.37	3.05	4.71	3.24	3.97	5.62	4.06	4.98	6.68	4.79	5.74	7.77
.350	1.38	1.89	3.23	2.38	3.07	4.73	3.23	3.97	5.65	4.10	5.02	6.74	4.85	5.76	7.80
.340	1.38	1.89	3.26	2.38	3.07	4.75	3.25	3.97	5.66	4.11	5.01	6.76	4.85	5.79	7.80
.320	1.39	1.90	3.29	2.39	3.06	4.74	3.27	3.99	5.62	4.14	5.02	6.84	4.87	5.83	7.83
.300	1.40	1.89	3.24	2.39	3.07	4.78	3.27	4.01	5.64	4.16	5.04	6.92	4.88	5.85	7.83
.280	1.40	1.90	3.20	2.40	3.08	4.79	3.29	4.01	5.66	4.19	5.07	6.88	4.89	5.82	7.87
.260	1.41	1.92	3.21	2.41	3.09	4.75	3.31	4.04	5.68	4.20	5.07	6.95	4.93	5.84	7.85
.250	1.41	1.91	3.23	2.42	3.08	4.75	3.32	4.04	5.68	4.21	5.07	6.96	4.94	5.85	7.85
.240	1.41	1.92	3.23	2.43	3.08	4.76	3.31	4.05	5.66	4.22	5.07	6.96	4.94	5.85	7.87
.220	1.41	1.93	3.21	2.43	3.08	4.75	3.33	4.06	5.66	4.23	5.08	6.95	4.98	5.88	7.90
.200	1.42	1.93	3.24	2.44	3.09	4.75	3.34	4.10	5.68	4.25	5.08	6.98	5.00	5.92	7.87
.180	1.43	1.94	3.23	2.45	3.11	4.76	3.34	4.10	5.67	1.28	5.13	7.01	5.03	5.94	7.88
.160	1.44	1.94	3.23	2.46	3.11	4.76	3.36	4.11	5.69	4.28	2.13	7.06	5.05	5.95	7.90
.150	1.43	1.94	3.23	2.46	3.13	4.76	3.35	4.11	5.69	4.29	5.13	7.06	5.05	5.97	7.90
.140	1.43	1.94	3.22	2.47	3.14	4.77	3.36	4.11	5.71	4.30	5.14	7.06	5.07	5.99	7.89
.120	1.44	1.95	3.18	2.48	3.15	4.81	3.38	4.11	5.67	4.30	5.14	7.06	5.08	6.02	7.88
.100	1.46	1.96	3.18	2.48	3.17	4.78	3.37	4.12	5.68	4.33	5.14	7.05	5.10	6.02	7.92
.080	1.47	1.96	3.21	2.49	3.18	4.75	3.38	4.13	5.66	4.34	5.16	7.07	5.11	6.03	7.94
.060	1.48	1.96	3.26	2.51	3.19	4.73	3.39	4.17	5.66	4.34	5.16	7.07	5.12	6.04	7.93
.050	1.48	1.98	3.26	2.52	3.19	4.73	3.41	4.17	5.66	4.37	5.19	7.08	5.13	6.04	7.92
.040	1.47	1.99	3.29	2.52	3.20	4.75	3.42	4.16	5.67	4.38	5.20	7.09	5.16	6.04	7.90
.030	1.47	1.99	3.29	2.53	3.21	4.76	3.43	4.19	5.68	4.41	5.19	7.10	5.17	6.05	7.91
.020	1.47	1.98	3.29	2.55	3.23	4.75	3.43	4.19	5.67	4.42	5.19	7.12	5.18	6.07	7.91

π_0	$q-p=6$			$q-p=7$			$q-p=8$			$q-p=9$			$q-p=10$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
.490	5.41	6.32	8.62	6.07	7.07	9.28	6.77	7.81	10.38	7.37	8.51	10.92	8.06	9.18	11.69
.480	5.41	6.37	8.58	6.10	7.09	9.28	6.80	7.86	10.39	7.41	8.60	10.98	8.07	9.21	11.75
.470	5.44	6.37	8.51	6.12	7.11	9.28	6.83	7.88	10.46	7.45	8.61	10.95	8.08	9.28	11.73
.460	5.47	6.43	8.58	6.12	7.12	9.30	6.84	7.91	10.41	7.47	8.65	10.99	8.12	9.31	11.76
.450	5.49	6.44	8.60	6.14	7.14	9.34	6.85	7.93	10.50	7.48	8.67	11.00	8.14	9.35	11.77
.440	5.49	6.46	8.59	6.16	7.15	9.32	6.88	7.97	10.51	7.51	8.70	11.02	8.15	9.40	11.78
.420	5.55	6.48	8.62	6.19	7.17	9.33	6.94	8.02	10.53	7.55	8.75	11.06	8.18	9.42	11.90
.400	5.56	6.52	8.59	6.24	7.22	9.33	6.96	8.06	10.48	7.61	8.78	11.08	8.25	9.42	11.88
.380	5.56	6.53	8.68	6.24	7.22	9.33	7.02	8.10	10.45	7.66	8.80	11.05	8.30	9.48	11.90
.360	5.60	6.55	8.70	6.29	7.23	9.41	7.06	8.14	10.45	7.70	8.84	11.06	8.33	9.50	11.94
.350	5.61	6.58	8.69	6.29	7.24	9.46	7.07	8.14	10.42	7.74	8.87	11.07	8.33	9.51	11.95
.340	5.61	6.59	8.66	6.29	7.26	9.52	7.09	8.13	10.45	7.75	8.88	11.10	8.35	9.53	11.97
.320	5.63	6.60	8.63	6.33	7.31	9.53	7.12	8.17	10.48	7.76	8.88	11.12	8.40	9.59	12.04
.300	5.64	6.63	8.65	6.37	7.34	9.54	7.14	8.17	10.49	7.81	8.92	11.08	8.45	9.61	11.98
.280	5.68	6.66	8.69	6.39	7.35	9.57	7.17	8.19	10.51	7.84	8.93	11.15	8.48	9.64	11.97
.260	5.71	6.68	8.69	6.41	7.39	9.55	7.17	8.23	10.58	7.87	8.97	11.17	8.52	9.66	12.01
.250	5.73	6.69	8.72	6.43	7.40	9.54	7.18	8.27	10.60	7.87	8.99	11.21	8.54	9.65	12.06
.240	5.75	6.71	8.76	6.42	7.40	9.54	7.19	8.29	10.64	7.90	8.99	11.22	8.56	9.66	12.08
.220	5.77	6.76	8.78	6.44	7.42	9.54	7.23	8.29	10.70	7.93	9.03	11.27	8.58	9.69	12.11
.200	5.79	6.77	8.79	6.46	7.46	9.56	7.25	8.32	10.71	7.97	9.03	11.27	8.63	9.73	12.14
.180	5.80	6.76	8.78	6.47	7.45	9.55	7.27	8.33	10.71	7.98	9.05	11.31	8.65	9.73	12.18
.160	5.82	6.76	8.79	6.50	7.48	9.57	7.32	8.35	10.75	8.03	9.08	11.36	8.71	9.76	12.16
.150	5.84	6.76	8.83	6.51	7.49	9.57	7.33	8.37	10.76	8.05	9.09	11.37	8.71	9.77	12.23
.140	5.84	6.77	8.86	6.50	7.50	9.61	7.35	8.40	10.74	8.08	9.10	11.38	8.72	9.78	12.24
.120	5.86	6.79	8.90	6.53	7.52	9.67	7.39	8.44	10.74	8.10	9.14	11.41	8.75	9.82	12.27
.100	5.88	6.81	8.87	6.57	7.56	9.69	7.42	8.47	10.76	8.13	9.16	11.40	8.80	9.86	12.29
.080	5.91	6.83	8.95	6.58	7.57	9.81	7.45	8.51	10.77	8.15	9.21	11.10	8.86	9.92	12.33
.060	5.94	6.85	8.98	6.61	7.57	9.84	7.49	8.54	10.81	8.19	9.23	11.38	8.90	9.97	12.39
.050	5.95	6.85	8.98	6.66	7.59	9.84	7.50	8.54	10.84	8.22	9.26	11.36	8.90	10.01	12.37
.040	5.96	6.89	9.01	6.68	7.62	9.93	7.52	8.54	10.84	8.23	9.27	11.36	8.93	10.04	12.41
.030	5.97	6.90	9.00	6.71	7.63	9.91	7.54	8.54	10.84	8.26	9.28	11.39	8.95	10.04	12.43
.020	5.99	6.91	8.99	6.73	7.67	9.92	7.58	8.59	10.83	8.29	9.28	11.38	8.98	10.06	12.47

Table 3

Asymptotic critical values for average mapping

π_0	$q-p=1$			$q-p=2$			$q-p=3$			$q-p=4$			$q-p=5$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
.490	2.65	3.73	6.34	4.58	5.89	9.10	6.24	7.71	11.17	7.81	9.58	13.30	9.27	11.10	15.24
.480	2.63	3.69	6.24	4.56	5.86	9.07	6.21	7.65	11.10	7.82	9.60	13.14	9.26	11.02	15.14
.470	2.64	3.67	6.20	4.54	5.83	9.08	6.19	7.62	11.09	7.80	9.55	13.09	9.20	10.97	15.10
.460	2.63	3.67	6.15	4.52	5.82	9.04	6.17	7.60	11.02	7.77	9.51	12.99	9.15	10.98	14.99
.450	2.61	3.63	6.15	4.51	5.78	9.05	6.14	7.61	10.93	7.76	9.50	12.93	9.15	10.94	14.90
.440	2.59	3.61	6.10	4.51	5.76	9.00	6.12	7.54	10.86	7.74	9.43	12.89	9.14	10.90	14.88
.420	2.58	3.59	6.09	4.48	5.71	8.99	6.05	7.45	10.68	7.74	9.38	12.71	9.07	10.90	14.72
.400	5.55	3.54	6.09	4.44	5.66	8.87	6.03	7.42	10.64	7.69	9.32	12.63	9.02	10.82	14.58
.380	2.54	3.52	6.09	4.40	5.61	8.76	5.99	7.35	10.52	7.63	9.25	12.50	8.98	10.67	14.40
.360	2.51	3.74	6.10	4.38	5.56	8.66	5.94	7.30	10.43	7.55	9.20	12.40	8.93	10.63	14.24
.350	2.50	3.45	6.05	4.35	5.54	8.64	5.93	7.27	10.34	7.52	9.19	12.38	8.91	10.58	14.21
.340	2.49	3.45	6.03	4.33	5.55	8.58	5.90	7.26	10.30	7.50	9.15	12.27	8.90	10.54	14.16
.320	2.47	3.43	5.91	4.27	5.53	8.55	5.87	7.20	10.14	7.48	9.08	12.21	8.83	10.47	14.02
.300	2.46	3.36	5.85	4.26	5.46	5.49	5.83	7.12	10.09	7.42	8.90	12.09	8.81	10.42	14.00
.280	2.45	3.32	5.82	4.22	5.43	8.38	5.79	7.07	9.96	7.37	8.88	11.99	8.76	10.37	13.81
.260	2.42	3.29	5.69	4.19	5.41	8.32	5.77	6.99	9.85	7.31	8.82	11.86	8.72	10.29	13.65
.250	2.42	3.29	5.65	4.17	5.39	8.27	5.75	6.97	9.78	7.30	8.78	11.87	8.68	10.23	13.63
.240	2.41	3.27	5.55	4.17	5.36	8.16	5.73	6.95	9.73	7.28	8.73	11.80	8.66	10.23	13.52
.220	2.39	3.25	5.48	4.16	5.30	8.07	5.70	6.89	9.58	7.23	8.65	11.73	8.60	10.15	13.35
.200	2.39	3.23	5.39	4.13	5.27	7.98	5.65	6.84	9.48	7.19	8.60	11.66	8.54	10.09	13.23
.180	2.36	3.19	5.30	4.10	5.22	7.91	5.60	6.79	9.34	7.15	8.56	11.58	8.50	10.04	13.12
.160	2.35	3.19	5.23	4.08	5.18	7.86	5.58	6.72	9.26	7.13	8.48	11.41	8.46	9.96	12.96
.150	2.33	3.18	5.18	4.07	5.14	7.85	5.56	6.70	9.18	7.12	8.44	11.35	8.46	9.92	12.88
.140	2.33	3.16	5.13	4.05	5.13	7.83	5.55	6.67	9.11	7.10	8.41	11.28	8.44	9.90	12.83
.120	2.31	3.13	5.10	4.02	5.10	7.78	5.52	6.64	9.01	7.04	8.36	11.15	8.39	9.83	12.75
.100	2.29	3.09	5.05	3.99	5.06	7.73	5.49	6.59	8.94	7.01	8.30	11.05	8.33	9.76	12.59
.080	2.28	3.05	4.98	3.96	5.01	7.62	5.44	6.54	8.86	6.95	8.24	10.95	8.27	9.70	12.44
.060	2.26	3.03	4.97	3.91	4.98	7.51	5.41	6.49	8.76	6.89	8.15	10.85	8.21	9.64	12.35
.050	2.26	3.01	4.91	3.90	4.95	7.46	5.39	6.47	8.74	6.86	8.10	10.80	8.20	9.59	12.30
.040	2.25	2.99	4.88	3.90	4.91	7.43	5.36	6.44	8.70	6.83	8.07	10.75	8.16	9.56	12.27
.030	2.24	2.98	4.84	3.89	4.89	7.33	5.34	6.42	8.64	6.81	8.04	10.69	8.13	9.23	12.19
.020	2.24	2.97	4.78	3.88	4.86	7.22	5.32	6.39	8.59	6.78	8.00	10.63	8.10	9.48	12.14

π_0	$q-p=6$			$q-p=7$			$q-p=8$			$q-p=9$			$q-p=10$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
.490	10.74	12.57	17.07	12.05	14.08	18.44	13.47	15.51	20.62	14.65	16.89	21.63	16.01	18.24	23.24
.480	10.70	12.56	16.91	12.04	14.01	18.33	13.37	15.52	20.43	14.62	16.86	21.65	15.93	18.20	23.17
.470	10.70	12.51	16.78	12.01	13.98	18.26	13.35	15.46	20.43	14.60	16.87	21.56	15.86	18.18	23.13
.460	10.67	12.51	16.72	11.97	13.91	18.15	13.31	15.44	20.43	14.60	16.87	21.56	15.82	18.19	23.05
.450	10.66	12.52	16.63	11.91	13.89	18.15	13.28	15.40	20.31	14.51	16.77	21.25	15.79	18.18	22.91
.440	10.62	12.43	16.52	11.89	13.79	18.04	13.26	15.32	20.23	14.47	16.68	21.22	15.76	18.12	22.77
.420	10.56	12.36	16.34	11.83	13.71	17.83	13.21	15.23	19.89	14.38	16.63	21.06	15.66	17.97	22.57
.400	10.49	12.35	16.16	11.77	13.62	17.64	13.17	15.12	19.66	14.36	16.55	20.84	15.59	17.85	22.46
.380	10.46	12.25	16.04	11.74	13.50	17.53	13.11	14.99	19.34	14.31	16.46	20.72	15.52	17.72	22.30
.360	10.39	12.22	15.98	11.68	13.40	17.36	13.06	14.93	19.05	14.28	16.36	20.57	15.47	17.59	22.29
.350	10.33	12.15	15.98	11.65	13.40	17.24	13.03	14.89	19.02	14.25	16.35	20.43	15.41	17.51	22.18
.340	10.30	12.12	15.95	11.59	13.36	17.22	13.00	14.81	18.89	14.22	16.29	20.34	15.36	17.48	22.11
.320	10.26	12.02	15.74	11.50	13.28	17.12	12.87	14.72	18.68	14.15	16.20	20.20	15.26	17.40	21.93
.300	10.18	11.95	15.65	11.44	13.21	17.05	12.81	14.63	18.66	14.10	16.11	20.11	15.19	17.33	21.66
.280	10.10	11.85	15.56	11.40	13.12	16.89	12.75	14.58	18.58	14.05	16.02	19.84	15.15	17.28	21.45
.260	10.06	11.80	15.44	11.36	13.05	16.68	12.68	14.53	18.44	13.98	15.95	19.71	15.09	17.16	21.34
.250	10.02	11.77	15.36	11.34	13.02	16.65	12.64	14.48	18.39	13.93	15.90	19.64	15.08	17.10	21.24
.240	9.98	11.73	15.23	11.29	12.99	16.65	12.61	14.45	18.29	13.86	15.84	19.61	15.03	17.02	21.19
.220	9.94	11.66	15.06	11.23	12.92	16.46	12.56	14.40	18.32	13.81	15.75	19.48	17.98	16.94	21.06
.200	9.89	11.58	14.89	11.17	12.84	16.40	12.48	14.34	18.09	13.73	15.65	19.30	14.91	16.85	20.92
.180	9.85	11.47	14.74	11.08	12.71	16.26	12.41	14.29	17.94	13.66	15.56	19.11	14.86	16.74	20.69
.160	9.81	11.39	14.62	11.03	12.63	16.16	12.38	14.24	17.80	13.57	15.48	18.91	14.76	16.60	20.43
.150	9.78	11.35	14.54	11.00	12.60	16.09	12.32	14.19	17.75	13.54	15.42	18.77	14.72	16.56	20.39
.140	9.75	11.33	14.50	10.96	12.60	16.01	12.30	14.14	17.71	13.51	15.37	18.66	14.69	16.53	20.32
.120	9.68	11.25	14.42	10.91	12.47	15.83	12.25	14.03	17.60	13.44	15.27	18.52	14.64	16.44	20.15
.100	9.63	11.16	14.30	10.85	12.38	15.67	12.20	13.92	17.40	13.38	15.16	18.36	14.57	16.33	20.00
.080	9.58	11.04	14.17	10.80	12.27	15.58	12.13	13.82	17.23	13.31	15.06	18.22	14.47	16.25	19.84
.060	9.54	11.00	14.03	10.73	12.17	15.49	12.05	13.71	17.05	13.23	14.94	18.08	14.36	16.15	19.70
.050	9.53	10.97	14.01	10.69	12.15	15.43	12.01	13.67	16.93	13.17	14.89	18.02	14.33	16.07	19.68
.040	9.48	10.93	13.93	10.65	12.12	15.34	11.98	13.63	16.86	13.14	14.82	18.01	14.30	16.02	19.56
.030	9.45	10.89	13.85	10.61	12.06	15.27	11.94	13.56	16.79	13.10	14.75	17.91	14.29	15.94	19.45
.020	9.42	10.85	13.84	10.57	12.00	15.18	11.90	13.50	16.75	13.05	14.69	17.77	14.28	15.90	19.38

Table 4: Level properties of parameter instability tests

T	ρ	LM	Wald	LR	PR1	PR2	PR3
			Ave	tests			
100	0	.036	.155	.215	.055	.126	.045
	.5	.030	.203	.260	.084	.156	.050
	.9	.022	.189	.231	.142	.149	.019
200	0	.045	.109	.131	.053	.090	.051
	.5	.041	.098	.115	.062	.089	.047
	.9	.018	.108	.126	.121	.092	.011
500	0	.043	.070	.075	.055	.068	.051
	.5	.053	.094	.102	.064	.083	.053
	.9	.036	.070	.078	.088	.070	.026
			Exp	tests			
100	0	.033	.253	.338	.065	.203	.051
	.5	.032	.318	.409	.114	.269	.052
	.9	.030	.323	.396	.208	.275	.014
200	0	.037	.159	.200	.056	.139	.047
	.5	.038	.158	.201	.065	.145	.037
	.9	.018	.171	.198	.178	.132	.010
500	0	.041	.079	.094	.050	.076	.047
	.5	.045	.103	.123	.067	.094	.050
	.9	.029	.097	.115	.132	.093	.015
			Sup	tests			
100	0	.027	.262	.356	.056	.216	.042
	.5	.025	.327	.446	.111	.287	.047
	.9	.045	.355	.438	.220	.310	.020
200	0	.028	.175	.218	.052	.154	.043
	.5	.030	.172	.231	.073	.165	.039
	.9	.027	.200	.234	.192	.179	.014
500	0	.037	.092	.112	.044	.089	.042
	.5	.040	.107	.127	.065	.107	.042
	.9	.031	.115	.136	.137	.114	.017

Notes: Entries to the Table are Monte Carlo simulations (1000 replications) of p-values for 5 % nominal tests.

Table 5: Level properties of parameter instability tests with $\hat{\sigma}^2 I_T$

T	ρ	LM	Wald	LR	PR1	PR2	PR3
			Ave	tests			
100	0	.038	.040	.040	.040	.042	.040
	.5	.037	.045	.045	.045	.037	.035
	.9	.031	.050	.050	.050	.008	.016
200	0	.033	.038	.038	.038	.035	.039
	.5	.028	.033	.033	.033	.027	.032
	.9	.023	.036	.036	.036	.008	.009
500	0	.053	.052	.052	.052	.055	.055
	.5	.045	.047	.047	.043	.045	.045
	.9	.036	.049	.049	.049	.025	.024
			Exp	tests			
100	0	.039	.040	.040	.040	.043	.044
	.5	.033	.039	.039	.039	.026	.030
	.9	.040	.058	.058	.058	.013	.013
200	0	037	.040	.040	.040	.039	.040
	.5	.040	.039	.039	.039	.037	.034
	.9	.031	.035	.035	.035	.004	.006
500	0	.055	.056	.056	.056	.058	.062
	.5	.040	.043	.043	.043	.041	.041
	.9	.033	.046	.046	.046	.016	.020
			Sup	tests			
100	0	.020	.027	.027	.027	.034	.036
	.5	.021	.023	.023	.023	.015	.020
	.9	.032	.049	.049	.049	.013	.017
200	0	.024	.027	.027	.027	.027	.032
	.5	.031	.033	.033	.033	.027	.031
	.9	.039	.048	.048	.048	.010	.015
500	0	.050	.052	.052	.052	.054	.061
	.5	.036	.037	.037	.037	.037	.036
	.9	.042	.050	.050	.050	.020	.023

Notes: Entries to the Table are Monte Carlo simulations (1000 replications) of p-values for 5 % nominal tests.

Table 6: Power properties of parameter instability tests for average mapping

T	ρ_1	ρ_2	δ	LM			PR1			PR3		
				s=.25	s=.50	s=.75	s=.25	s=.50	s=.75	s=.25	s=.50	s=.75
100	0	.10	.10	.044	.052	.039	.071	.082	.069	.060	.068	.059
	0	.25	.25	.100	.146	.080	.148	.221	.149	.135	.172	.114
	0	.50	.50	.366	.437	.199	.477	.582	.461	.440	.500	.297
	.50	.60	.10	.053	.054	.035	.160	.170	.127	.062	.059	.037
	.50	.25	.25	.071	.125	.095	.061	.090	.071	.119	.165	.122
	.50	0	.50	.223	.420	.251	.155	.334	.213	.318	.509	.351
	.90	.80	.10	.013	.030	.047	.078	.047	.066	.019	.029	.045
	.90	.65	.25	.038	.150	.152	.047	.019	.021	.055	.145	.138
	.90	.40	.50	.110	.931	.356	041	.034	.053	.190	.508	.466
200	0	.10	.10	.066	.071	.059	.085	.087	.074	.082	.079	.070
	0	.25	.25	.188	.286	.157	.240	.330	.216	.211	.300	.179
	0	.50	.50	.639	.829	.442	.675	.894	.680	.687	.857	.515
	.50	.60	.10	.077	.098	.055	.129	.195	.128	.077	.096	.054
	.50	.25	.25	.158	.294	.225	.118	.230	.144	.193	.334	.250
	.50	0	.50	.501	.788	.562	.439	.736	.560	.590	.848	.664
	.90	.80	.10	.047	.111	.122	.043	.016	.021	.039	.093	.093
	.90	.65	.25	.144	.437	.442	.043	.045	.043	.159	.419	.389
	.90	.40	.50	.288	.764	.689	.066	.173	.193	.466	.867	.826
500	0	.10	.10	.106	.148	.109	.116	.163	.118	.117	.160	.112
	0	.25	.25	.456	.671	.399	.487	.705	.450	.483	.686	.430
	0	.50	.50	.960	.998	.928	.964	1.000	.979	.972	.999	.951
	.50	.60	.10	.150	.203	.104	.221	.298	.182	.145	.198	.105
	.50	.25	.25	.408	.712	.547	.361	.655	.452	.431	.733	.587
	.50	0	.50	.930	.999	.940	.896	.997	.950	.951	.999	.966
	.90	.80	.10	.121	.364	.328	.025	.069	.049	.111	.344	.291
	.90	.65	.25	.424	.886	.840	.130	.438	.308	.445	.878	.854
	.90	.40	.50	.661	.980	.879	.329	.754	.845	.879	1.00	.996

Notes: Entries to the Table are Monte Carlo simulations (1000 replications) of p-values for 5 % nominal tests.

Table 7: Power properties of parameter instability tests for exponential mapping

T	ρ_1	ρ_2	δ	LM			PR1			PR3		
				s=.25	s=.50	s=.75	s=.25	s=.50	s=.75	s=.25	s=.50	s=.75
100	0	.10	.10	.039	.043	.037	.083	.091	.091	.054	.062	.065
	0	.25	.25	.089	.120	.078	.153	.226	.181	.126	.157	.126
	0	.50	.50	.389	.383	.191	.477	.601	.562	.459	.493	.338
	.50	.60	.10	.051	.045	.026	.200	.208	.201	.058	.057	.032
	.50	.25	.25	.052	.108	.092	.056	.077	.052	.099	.157	.127
	.50	0	.50	.192	.371	.267	.147	.260	.200	.307	.488	.395
	.90	.80	.10	.024	.044	.072	.151	.096	.103	.019	.027	.037
	.90	.65	.25	.046	.153	.227	.100	.036	.023	.046	.109	.131
	.90	.40	.50	.096	.369	.479	.076	.023	.025	.187	.452	.465
200	0	.10	.10	.066	.053	.046	.093	.088	.065	.092	.076	.067
	0	.25	.25	.199	.255	.142	.258	.320	.243	.221	.290	.199
	0	.50	.50	.684	.815	.440	.706	.896	.790	.735	.858	.588
	.50	.60	.10	.080	.075	.051	.129	.220	.182	.077	.088	.047
	.50	.25	.25	.149	.284	.228	.105	.161	.111	.189	.316	.265
	.50	0	.50	.528	.788	.533	.417	.683	.529	.633	.847	.729
	.90	.80	.10	.042	.104	.156	.084	.035	.036	.040	.068	.094
	.90	.65	.25	.112	.409	.499	.054	.029	.017	.142	.353	.406
	.90	.40	.50	.268	.756	.772	.051	.059	.094	.523	.848	.862
500	0	.10	.10	.112	.144	.111	.125	.162	.131	.127	.155	.127
	0	.25	.25	.484	.647	.449	.507	.685	.527	.500	.670	.491
	0	.50	.50	.984	.997	.962	.976	1.000	.995	.984	1.000	.973
	.50	.60	.10	.158	.179	.097	.257	.313	.226	.154	.198	.107
	.50	.25	.25	.421	.697	.570	.351	.595	.435	.478	.725	.625
	.50	0	.50	.956	.998	.978	.935	.996	.977	.968	1.00	.989
	.90	.80	.10	.105	.350	.352	.043	.022	.009	.080	.286	.294
	.90	.65	.25	.394	.890	.889	.075	.222	.122	.440	.864	.881
	.90	.40	.50	.759	.982	.929	.268	.520	.633	.941	.999	.998

Notes: Entries to the Table are Monte Carlo simulations (1000 replications) of p-values for 5 % nominal tests.

Table 8: Power properties of parameter instability tests for supremum mapping

T	ρ_1	ρ_2	δ	LM			PR1			PR3		
				s=.25	s=.50	s=.75	s=.25	s=.50	s=.75	s=.25	s=.50	s=.75
100	0	.10	.10	.028	.029	.036	.067	.080	.087	.047	.041	.060
	0	.25	.25	.055	.069	.049	.138	.193	.166	.088	.099	.096
	0	.50	.50	.280	.223	.099	.430	.541	.536	.353	.348	.278
	.50	.60	.10	.037	.032	.015	.191	.201	.210	.044	.036	.027
	.50	.25	.25	.037	.074	.077	.055	.061	.054	.062	.111	.104
	.50	0	.50	.114	.219	.199	.119	.167	.145	.229	.339	.333
	.90	.80	.10	.043	.066	.094	.163	.113	.199	.030	.035	.044
	.90	.65	.25	.067	.158	.262	.116	.051	.024	.046	.092	.132
	.90	.40	.50	.114	.345	.473	.084	.022	.011	.154	.327	.382
200	0	.10	.10	.046	.040	.034	.078	.077	.060	.064	.060	.051
	0	.25	.25	.154	.171	.095	.224	.269	.222	.179	.203	.146
	0	.50	.50	.605	.685	.336	.663	.846	.769	.653	.758	.526
	.50	.60	.10	.067	.058	.030	.170	.226	.189	.052	.061	.031
	.50	.25	.25	.120	.202	.197	.082	.093	.067	.154	.241	.245
	.50	0	.50	.423	.684	.533	.316	.492	.405	.565	.761	.700
	.90	.80	.10	.063	.124	.187	.106	.043	.038	.042	.050	.109
	.90	.65	.25	.116	.386	.515	.060	.025	.013	.120	.272	.400
	.90	.40	.50	.254	.704	.761	.052	.010	.031	.466	.749	.827
500	0	.10	.10	.091	.108	.091	.122	.141	.118	.104	.129	.107
	0	.25	.25	.440	.550	.403	.467	.620	.512	.448	.566	.464
	0	.50	.50	.976	.993	.949	.969	.999	.994	.980	.998	.974
	.50	.60	.10	.143	.128	..072	.255	.304	.236	.131	.177	.072
	.50	.25	.25	.367	.610	.565	.278	.446	.324	.478	.653	.611
	.50	0	.50	.944	.994	.968	.897	.987	.959	.962	.998	.988
	.90	.80	.10	.106	.330	.392	.048	.013	.009	.082	.242	.319
	.90	.65	.25	.369	.851	.901	.043	.047	.009	.392	.800	.874
	.90	.40	.50	.730	.976	.915	.169	.165	.178	.929	.996	.998

Notes: Entries to the Table are Monte Carlo simulations (1000 replications) of p-values for 5 % nominal tests.